

# Ph 77 ADVANCED PHYSICS LABORATORY

## – ATOMIC AND OPTICAL PHYSICS –

### Interferometric Measurement of Resonant Absorption and Refractive Index in Rubidium

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#### I. BACKGROUND

In this lab you will observe the relation between resonant absorption and the refractive index in rubidium gas. To see how these are related, consider a simple model for a rubidium atom, namely that of a single electron bound by a harmonic force, acted upon by the electric field of an incident laser (see for example Jackson 1975, pg. 284, Marion and Heald 1980, pg. 282). Although crude, this model does allow us to write down the basic optical properties of a gas of atoms near an atomic resonance. In this picture, the equation of motion for the electron around the atom is

$$m[\ddot{x} + \gamma\dot{x} + \omega_0^2x] = -eE(x, t)$$

where  $\gamma$  measures a phenomenological damping force. If the electric field varies in time as  $Ee^{-i\omega t}$ , then the dipole moment contributed by one atom is

$$\begin{aligned} p &= -ex \\ &= (e^2/m)(\omega_0^2 - \omega^2 - i\omega\gamma)^{-1}E \\ &= \epsilon_0\chi_e E \end{aligned}$$

where  $\chi_e$  is called the electric susceptibility. If there are  $N$  atoms per unit volume, then the (complex) dielectric constant of the gas is given by

$$\begin{aligned} \epsilon(\omega)/\epsilon_0 &= 1 + 4\pi\chi_e \\ &= 1 + \frac{4\pi N f e^2/m}{(\omega_0^2 - \omega^2 - i\omega\gamma)} \end{aligned} \tag{1}$$

where  $f$  is a standard fudge factor, called the “oscillator strength” of the transition. Adding the oscillator strength factor makes this simple classical calculation agree with a more realistic quantum mechanical calculation. The oscillator strength is of order unity for strong transitions like the  $S \rightarrow P$  rubidium lines, and is much smaller for forbidden atomic transitions. Both the oscillator strength and the damping factor  $\gamma$  are difficult to calculate for real atoms, since doing so requires quite a lot of detailed atomic physics.

Maxwell’s equations (MKS units) for a propagating electromagnetic wave give us

$$\nabla^2 E - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

and we define an index of refraction  $n = c/v = \sqrt{\epsilon\mu/\epsilon_0\mu_0}$ , where  $v$  is the speed of wave propagation. Assuming  $\mu/\mu_0 \simeq 1$  and the above expression for the dielectric constant  $\epsilon/\epsilon_0$ , we find ourselves with a complex index of refraction, which we write

$$n = \sqrt{\epsilon/\epsilon_0} = n_0(1 + i\kappa) \tag{2}$$

where  $n_0$  and  $\kappa$  are real quantities. Evaluating Eqn. 1 gives

$$Re(\sqrt{\epsilon/\epsilon_0}) = n_0$$

$$\begin{aligned}
&\simeq 1 - \frac{2\pi(\omega^2 - \omega_0^2)Nfe^2/m}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2} \\
&\simeq 1 - \frac{\pi\Delta\omega Nfe^2/m\omega_0}{\Delta\omega^2 + \gamma^2/4} \\
\text{Im}(\sqrt{\epsilon/\epsilon_0}) &= n_0\kappa \\
&\simeq \frac{2\pi Nf\omega\gamma e^2/m}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2} \\
&\simeq \frac{\pi Nf\gamma e^2/2m\omega_0}{\Delta\omega^2 + \gamma^2/4}
\end{aligned}$$

where  $\Delta\omega = \omega - \omega_0$ . These are plotted in Figure 1. This is the index of refraction for a dilute atomic gas, which of course is proportional to the atom density.

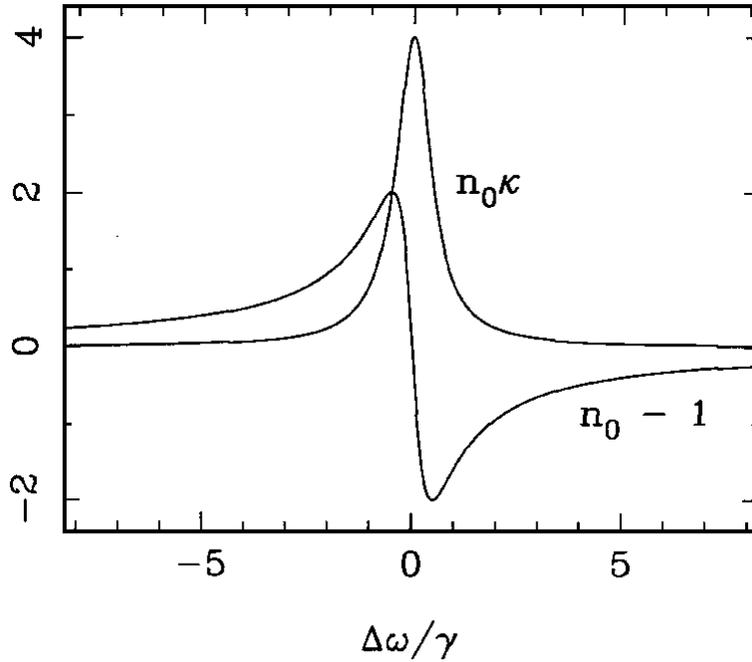


Figure 1. Plot of the absorption  $n_0\kappa$  and refractive index change  $n_0 - 1$  for a gas near an atomic resonance. Note the index change is proportional to the first derivative of the absorption.

An electromagnetic wave in the medium propagates according to

$$e^{-i(\omega t - nkz)} = e^{-kn_0\kappa z} e^{-i[\omega t - kn_0 z]} \quad (3)$$

where  $k = \omega/c$ . From this it can be seen that  $n_0$  corresponds to the usual index of refraction, equal to  $c/v$ , while  $\kappa$  describes the attenuation of the wave. Note that a relation  $n_0 - 1 \simeq -2\Delta\omega\kappa/\gamma$  exists between the index of refraction and the attenuation, which is independent of the oscillator strength of the atomic transition. This relation, showing that  $n_0(\nu)$  and  $\kappa(\nu)$  can be derived from one another, is an example of the more general *Kramers-Kronig relations*. A full quantum mechanical treatment also yields the same relation for the absorption and refractive index of a gas near an atomic resonance. The goal of this lab

is to measure both the absorption and index of refraction variations of rubidium gas around the  $S \rightarrow P$  resonance lines. While absorption is easy to observe, refractive index changes are not, so an interferometric technique will be used to observe it.

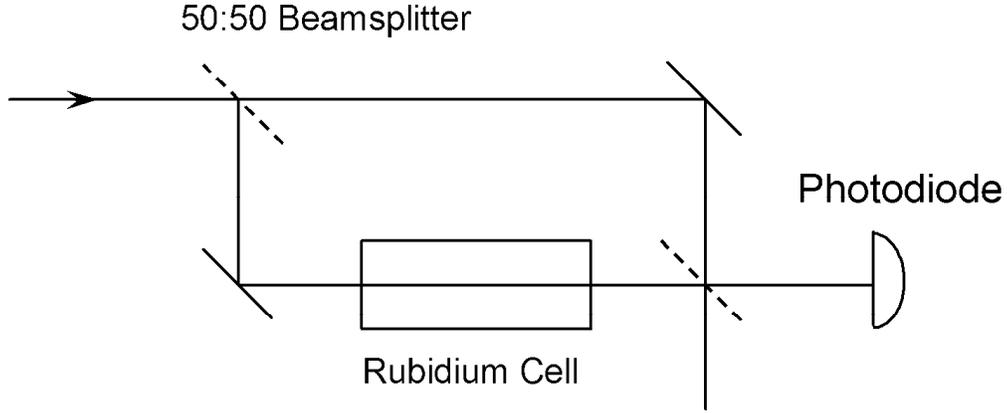


Figure 2. The basic experimental set-up, consisting of a rubidium vapor cell in one arm of a Mach-Zehnder interferometer. The dotted lines represent 50:50 beamsplitters. The input laser scans across the (Doppler broadened) rubidium absorption line.

Let us examine the experimental set-up shown in Figure 2, consisting of a rubidium vapor cell in one arm of a Mach-Zehnder interferometer. The Mach-Zehnder interferometer is related to the Michelson interferometer, with which you are probably familiar. The input laser light is first split by a beamsplitter (we will assume both beamsplitters in the interferometer are perfect lossless 50:50 beamsplitters), and the two beams travel down different paths through the interferometer. They are recombined at the second beamsplitter, and the light intensity in one direction is measured with a photodetector. The intensity seen at the photodiode is sensitive to the relative phases of the two beams as they interfere at the second beamsplitter.

Your first job, *before* attempting the experiment, is to model the expected signal seen at the photodiode in Figure 2, as the laser frequency is scanned through the rubidium resonance line. If we consider the interferometer in Figure 2 without the rubidium cell, it is straightforward to calculate the photodiode signal. As the two beams propagate through the separate arms of the interferometer, each picks up a phase shift as it travels, given in Eqn. 3. Without the rubidium cell  $n_0 = 1$  (neglecting the contribution from  $n_{air}$ ) and  $\kappa = 0$ , giving simple free-space propagation  $e^{ikz}$ .

The output power hitting the photodiode comes from the combination of the two beams at the second beamsplitter, and is given by

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{4} |e^{ikL_1} + e^{ikL_2}|^2 \\ &= [1 + \cos(k\Delta L)]/2 \end{aligned}$$

which is plotted in Figure 3. Since the beam splitters are perfect 50:50 beamsplitters, the beams in the two paths have equal intensity, so the photodiode output as a function of  $\Delta L = L_2 - L_1$  varies from zero (destructive interference) to the initial laser intensity  $I_0$  (constructive interference) as shown in the figure.

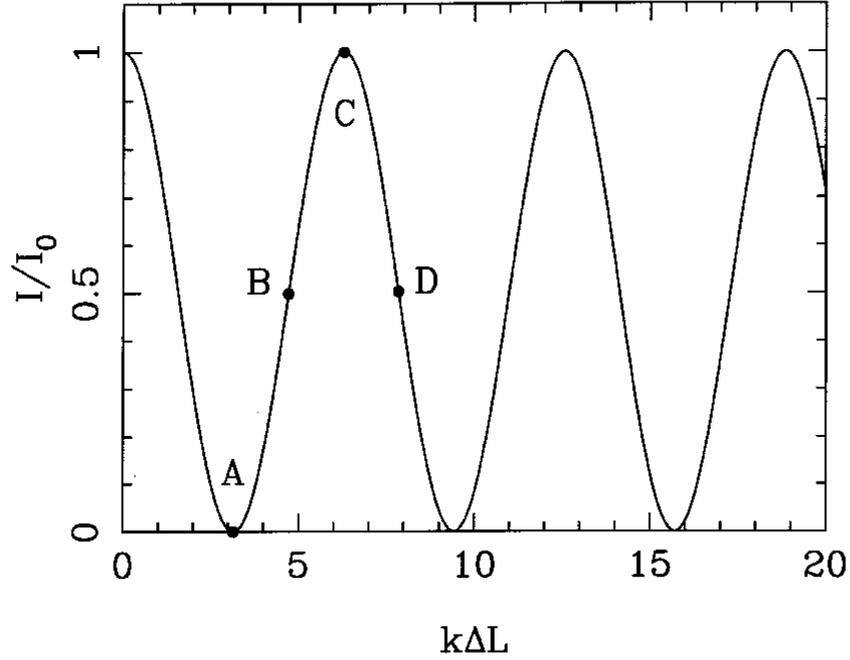


Figure 3. Photodiode output vs.  $k\Delta L$ , where  $k = \omega/c = 2\pi/\lambda$ , for a perfect Mach-Zehnder interferometer with no rubidium cell, at fixed laser frequency.

Next consider the effect of the rubidium cell on the propagation of a laser. From Eqn. 3, the total phase shift upon passing through the cell is

$$\begin{aligned} e^{-kn_0\kappa\Delta z} e^{ikn_0\Delta z} &= e^{-kn_0\kappa\Delta z} e^{ik\Delta z} e^{ik(n_0-1)\Delta z} \\ &= e^{-\tau} e^{ik\Delta z} e^{i\delta} \end{aligned}$$

where  $\Delta z$  is the length of the cell. The factor  $e^{ik\Delta z}$  in this expression is the free-space propagation factor. The  $e^{-\tau}$  factor comes from attenuation in the cell, with  $\tau = kn_0\kappa\Delta z \approx k\kappa\Delta z$ . Because we have a resonance line,  $\tau$  depends on frequency and we can assume a Lorentzian line profile,

$$\tau = \frac{\tau_0\gamma^2}{\Delta\omega^2 + \gamma^2}$$

where  $\tau_0$  is the absorption at line center. The  $e^{i\delta}$  factor is the additional phase shift from the refractive index of the rubidium atoms, with  $\delta = k(n_0 - 1)\Delta z$ . The atomic factors are related through  $\delta = (n_0 - 1)\tau/\kappa = -2\Delta\omega\tau/\gamma$ , which you should verify.

If we now put the rubidium cell in the interferometer, the photodiode output will be given by

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{4} |e^{ikL_1} + e^{-\tau} e^{ikL_2} e^{i\delta}|^2 \\ &= [1 + e^{-2\tau} + 2e^{-\tau} \cos(k\Delta L + \delta)]/4. \end{aligned}$$

Note that if the rubidium density is zero, then  $\tau = \delta = 0$  and we have the same result as before. Note also that three terms in this equation are frequency dependent:  $\tau$ ,  $\delta$ , and  $k$ . However, if  $\Delta L$  is small then  $k\Delta L$  changes very little as the laser frequency is scanned over a rubidium line, so we can assume  $k\Delta L$  is

essentially constant as a function of laser frequency (see Problem 1).

**Problem 1.** Consider the photodiode output from the interferometer without the rubidium cell. Figure 3 shows the output at fixed laser frequency as a function of  $\Delta L$ . The maxima in this are referred to as “fringes,” from their spatial structure (which you will see in the lab when you set up the interferometer). How small must  $\Delta L$  be in order for the photodiode output to go through less than one fringe as the laser is scanned over the rubidium resonance line (call it 5 GHz)? To get the best results, you should try to set up your interferometer with  $\Delta L$  less than this.

**Problem 2.** Compute the photodiode output as a function of laser frequency around the rubidium resonance line,  $I(\Delta\omega)/I_0$ , for the set-up shown in Figure 2. Assume the atoms in your cell are at rest (for ease of calculation) with some linewidth  $\gamma$ , so we can use the Lorentzian profile above for  $\tau(\omega)$ . Make three different plots of  $I(\Delta\omega)/I_0$ , one for each of three different values of the line-center optical depth:  $\tau_0 = 0.4$ , 2, and 20. Make your plots over the range  $-20\gamma < \Delta\omega < 20\gamma$ . Plot six curves on each plot, with values of  $k\Delta L \bmod(2\pi)$  equal to  $j\pi/5$ , with  $j = 0$  to 5. The first and last of these correspond to the positions A and C in Figure 3. Label your plots. You will be trying to reproduce these curves in the lab. (Check your calculations by comparing with the one calculated curve in Figure 5 below.) Why does  $I(\Delta\omega = 0)/I_0$  go to 0.25 for large  $\tau_0$ ?

Amazingly enough, the generality of the Kramers-Kronig relations says that the above calculations relating  $n_0 - 1$  and  $\kappa$  are true for a Doppler-broadened gas as well as for atoms at rest. Because of this, scanning the laser over the Gaussian profile of the Doppler-broadened gas will give results which are qualitatively similar to those you calculated in Problem 2 for the atoms’ natural Lorentzian profile.

## II. LABORATORY EXERCISES.

**The Clausius-Clapeyron Relation.** Before launching into the main part of the lab, we’ll get warmed up by first measuring just the resonant absorption as a function of the rubidium cell temperature. Heating the cell increases the rubidium vapor density and thus increases the absorption. The rubidium vapor in the cell is in equilibrium with a small bit of solid rubidium on the cell wall, and the vapor pressure is given by the Clausius-Clapeyron relation

$$\begin{aligned} p(T) &= p_0 e^{-L/RT} \\ &= p_0 e^{-\ell/kT} \end{aligned}$$

where  $p_0$  is a constant,  $T$  is the cell temperature in Kelvin,  $L$  is the latent heat of vaporization per mole,  $\ell$  is the latent heat per atom,  $R$  is the gas constant, and  $k$  is Boltzmann’s constant. This equation is derived from rather fundamental thermodynamic relations, but the derivation is a bit too involved to repeat here. Most good books on statistical mechanics derive it. For example, you can find it in Reif’s book (see references below), which is still an excellent introduction to the subject.

Assuming the rubidium gas behaves like an ideal gas (a good assumption), the vapor density is proportional to  $e^{-\ell/kT}$ , and thus so is the optical depth  $\tau(\omega)$ . The light transmitted through the cell is equal to  $I_{out}(\omega) = I_{in}e^{-\tau(\omega)}$  in the limit that  $I_{in}$ , the light incident on the cell, is much less than the saturation intensity (which was introduced in the previous lab, equal to about 2 mW/cm<sup>2</sup> for rubidium). Thus we have

$$\frac{I_{out}(\omega)}{I_{in}} = \exp[-A(\omega) \exp(-\ell/kT)]$$

where the function  $A(\omega)$  contains the Doppler-broadened absorption profile of the gas. If we measure the

intensity of the line center only, then

$$\frac{I_{out}(\omega_0)}{I_{in}} = \exp[-A_0 \exp(-\ell/kT)]$$

where here  $A_0$  is a constant for a given atomic transition. The goal of the first part of the lab will be to measure  $I_{out}(\omega_0)/I_{in}$  at several different values of the cell temperature, and from these data extract the latent heat of vaporization of rubidium gas.

Start with the cell at room temperature (about 25C on the cell temperature controller). Scan the laser frequency and send the beam through the rubidium cell and onto a photodiode. Reduce the laser intensity by about 3-4 orders of magnitude by using absorption filters, in order to reduce the intensity well below the saturation value. Tune the laser so you can see all four of the rubidium transitions, although probably not all in a single sweep. Check how much background light is getting into the photodiode by blocking the laser beam. You may need to turn out the lights and shield the photodetector to keep the stray light down. Remember that zero volts on the photodiode may not mean zero light. All amplifiers have offsets, so you may need to compensate for the photodiode reading at zero light.

When you observe the photodiode signal on the 'scope, you will probably notice that  $I_{out}(\omega_0)/I_{in}$  changes with laser settings, in particular with the laser current. This is because the laser doesn't always run in a single mode. When it runs multi-mode, some light is not resonant with the atoms and thus is not absorbed. This is a serious problem that limits how accurately you can measure  $I_{out}(\omega_0)/I_{in}$ . You can get pretty good results if you do the following: set the high voltage so the transition you want to observe is centered in the sweep, and then adjust the laser current to *minimize*  $I_{out}(\omega_0)/I_{in}$ .

If you think the laser is scanning okay and giving you accurate measurements, then start making measurements of  $I_{out}(\omega_0)/I_{in}$  on the middle **85a** line as a function of temperature. Measure  $I_{out}(\omega_0)$ ,  $I_{out}(nonresonant) \approx I_{in}$  and  $I_{dark}$  at each temperature. You can take  $I_{out}(nonresonant)$  to be an eyeball average of the intensity on either side of the line. Don't move the cell or any of the optics during the measurements; only adjust the laser settings a small amount in order to minimize  $I_{out}(\omega_0)$ . Make sure you measure especially carefully when  $I_{out}(\omega_0)/I_{in}$  is small. Measure at temperatures from 25C to 75C in increments of 5-10C. You don't need to wait a long time to reach some particular temperature exactly; just make sure the temperature is fairly stable for each reading. The temperature is stable enough if it changes by less than 0.1C in 10 seconds.

When you have the data, you should get a straight line when you plot  $\log(\log(I_{out}(\omega_0)/I_{in}))$  versus  $1/T$  (why? - see above). Extract the latent heat of vaporation from the slope of this line. Express your answer in Joules/gram.

Also, plot  $I_{out}(\omega_0)/I_{in}$  as a function of temperature, along with a curve going through the data using the Clausius-Clapeyron relation with the parameters you measured. If you plot the fit from about  $T = -10C$  to  $T = 80C$  you can also see the low-temperature structure of the absorption versus temperature.

**The Kramers-Kronig Relation.** Next, move on to the main event of observing the Kramers-Kronig relation in the lab by measuring Mach-Zehnder spectra like those you calculated in Problem 2. The first thing you should do in the lab is check your calculations with your TA. If your calculations aren't right, the lab will make no sense at all.

The optical set-up is shown in Figure 4. To begin the lab work, set up the Mach-Zehnder interferometer using the diode laser, just to get a feeling for what the fringe pattern looks like. Follow the set-up in Figure 4, starting out without the negative lens and without the neutral-density (ND) filter. Make sure the beam goes through the centers of the rubidium cell windows, where the optical quality is best. In keeping with

what you found in Problem 1 above, make sure the two arms of the interferometer are about the same length. Check that the laser is tuned on resonance by blocking one arm of the interferometer and putting the ND filter back in. You should then see a nice absorption spectrum on the photodiode when you scan the laser.

Adjust the mirrors so the two beams overlap on the second beamsplitter, and then adjust the second beamsplitter so the two beams are collinear. If the beams overlap well at the beamsplitter, and they also overlap some distance downstream from the beamsplitter, then you know they must overlap everywhere. Iterate these steps so the two beams are overlapping and collinear as best you can. At this point you should start to see fringes on the interfering beams. Put in the negative lens to expand the beam before it hits the photodiode. This makes it easier to see the fringe pattern, and you can adjust the interferometer so that broad fringes are seen. They should be broad enough so that the photodiode only samples a small part of a fringe.

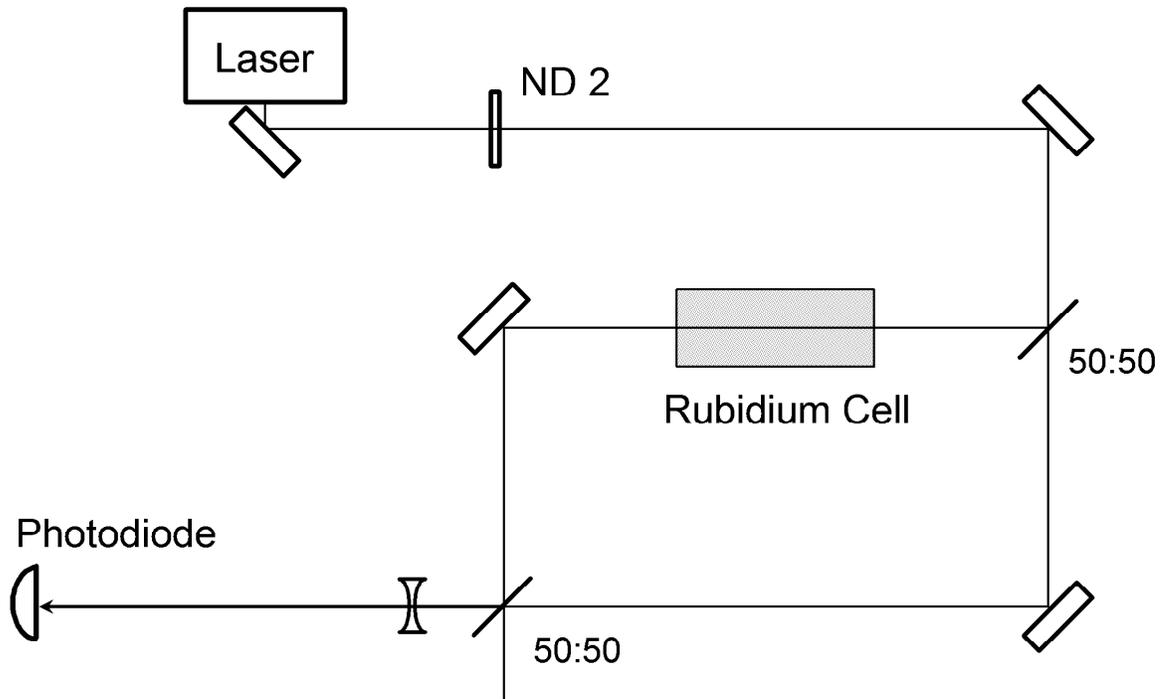


Figure 4. Optical layout for the main part of the lab. The ND filter should usually be removed when aligning the beams. Point the TV camera at the photodiode when looking at fringes.

You should also note that by gently pressing on the breadboard one can move the fringe pattern (effectively changing  $\Delta L$  above). With the photodiode sampling the interfering beams and the laser off resonance, wiggle one of the mirrors with your finger (gently!) while watching the photodiode output on the oscilloscope. You should see a (time-dependent) fringe pattern that looks something like that shown in Figure 3. Measure the fringe contrast,  $(I_{\max} - I_{\min})/I_{\max}$ . You can adjust the interferometer while wiggling the mirror to get high contrast fringes on the oscilloscope. You may find it necessary to play with the alignment a bit to get a good fringe contrast. For best results the contrast should be better than 0.8,

since your theory assumed a contrast of unity. When you get good fringes, capture the photodiode output (while wiggling the mirror) on the digital 'scope, and put a hard copy in your notebook.

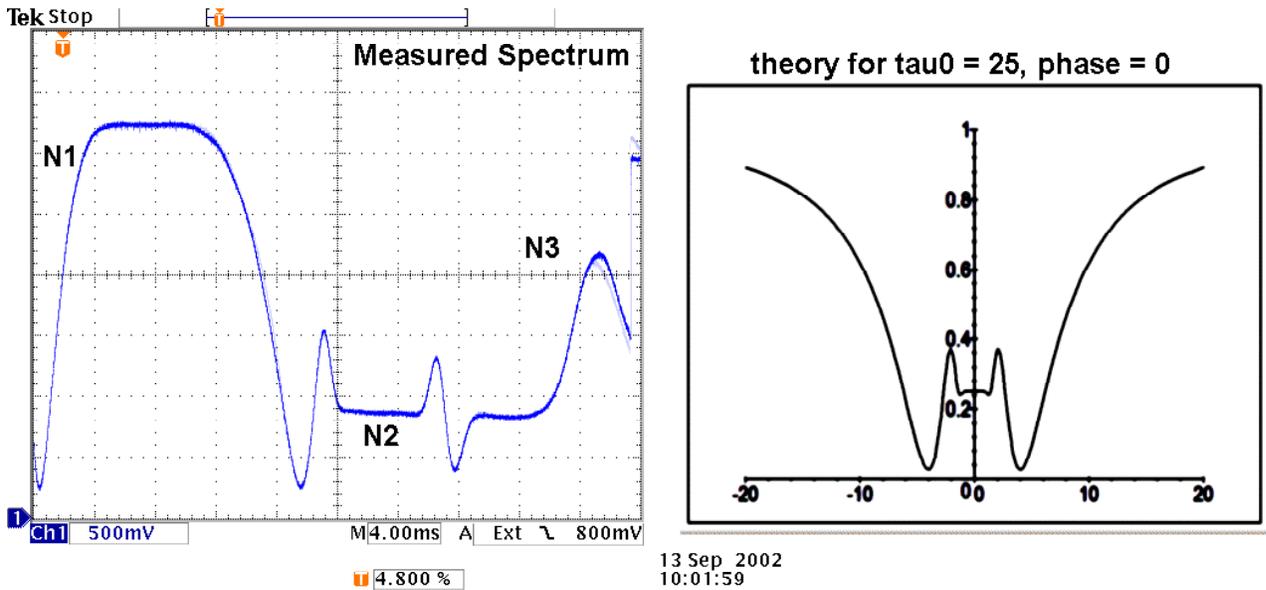


Figure 5. A comparison of a measured spectrum (left) with a calculated spectrum (right). The plot shows  $I(\Delta\omega)/I_0$  versus  $\Delta\omega/\gamma$ . The calculation assumed  $\tau = 25$  at line center and  $k\Delta L=0$ . The measured spectrum is for the 85b line, but the adjacent 87b line complicates the right side of the spectrum (marked by N3). The center of the 85b line is at N2. The feature at N1 is an artifact of the laser scanning.

Next block the arm of the interferometer without the rubidium cell, in order to observe the rubidium absorption line without any interferometer effects. If all is going well, you should see a nice strong Doppler-broadened absorption line, without any serious mode hops. The ND filter is necessary to avoid saturating the line (which makes it broader). Tune the laser to get a nice strong 85b line, with the 87b line on the side. Have your TA check it out, and save a spectrum.

Now unblock the second arm of the interferometer, and watch the oscilloscope. As you push on the optical bench, you can see different points in the interferometer fringe pattern, and you should see an output something like what you calculated in Problem 2 for low  $\tau$ . Play around with the interferometer until you understand what's going on and your spectra agree reasonably well with theory. Have your TA take a look at the spectra to see that everything looks good. Capture three good traces, corresponding roughly to points B, C, and D in Figure 3.

Lastly, heat the rubidium cell by turning the controller setting to 100C. Watch the spectra as the cell heats up. It will take about 15 minutes, but then you should begin seeing spectra that look like what you calculated for high  $\tau$ . Figure 5 shows some typical results for one phase. The data will probably not be a perfect match to calculation, but the results should provide a reasonable demonstration of the Kramers-Kronig relations.

Take several spectra at high  $\tau$ , at different phase angles. In particular, take spectra at  $k\Delta L=0$  and  $k\Delta L=\pi$ .

### III. REFERENCES.

Jackson, J. D. 1975, *Classical Electrodynamics*, 2nd Edition.

Marion, J. B. and Heald, M. A. 1980, *Classical Electromagnetic Radiation*, 2nd Edition.

Reif, F, 1965, *Fundamentals of Statistical and Thermal Physics*.