

Ph 3 - INTRODUCTORY PHYSICS LABORATORY

– California Institute of Technology –

Laser Interferometry

1 Introduction

The purpose of this lab is for you to learn about the physics of laser interferometry and the science of precision measurements, and for you to gain some laboratory experience with optics and electronic test equipment along the way. You will build and align a table-top laser interferometer in the Ph3 lab, you will measure the small motions of an oscillating mechanical system using the interferometer, and you will examine how accurately you can measure distance using this interferometer.

1.1 Laser Interferometry

In this experiment you will be building a basic Michelson interferometer (named after Albert Michelson, who invented it in the late nineteenth century). While optical interferometry can be done using broad-band light sources (as Michelson did long before lasers were invented), it is much easier with a nearly monochromatic light source like a laser. You will be using the optical layout shown in Figure 1.

The basic idea is fairly simple: the initial laser beam is split into two equal-intensity beams by the beam splitter, these beams both reflect off their respective end mirrors, and the beams recombine once again at the beamsplitter. How much light ends up going into the detector depends on the relative phase of the recombining beams. If the beams are in phase, then we say they combine *constructively*, and in this case the beams add and the full initial laser beam intensity lands on the detector. But if the beams are out of phase, then they combine *destructively*, so no light lands on the detector. If we translate one of the end mirrors as shown in Figure 1, then small changes in the mirror position can be measured by the changing light intensity on the detector. Laser interferometry is a sensitive way of measuring small displacements, and it is a non-contact measurement that can operate over quite large distances.

To understand how the interferometer works in detail, we have to look at the math. If we approximate our Gaussian laser beams as plane waves, then we can write the electric field of a propagating light wave as

$$E(z, t) = E_0 e^{i(\omega t - kz)}$$

where E_0 is a constant, $\omega = 2\pi c/\lambda$ is the angular frequency of the light, $k = 2\pi/\lambda$ is the wave number, and λ is the light wavelength. Here we have defined our coordinates so that the laser beam is propagating in the z direction. You will be using a Helium-Neon laser, which puts out red light with $\lambda = 633$ nm, so the oscillation frequency of the electric field is $\nu = \omega/2\pi = c/\lambda \approx 4.7 \times 10^{14}$ Hz.

Since we have no electronics that can operate at 470 THz, we do not measure any electric fields when we are working with visible light. Instead we measure the power in a beam, which is proportional to the square of the electric field averaged over time, and we write this as $I_0 = CE_0^2$ where C is a proportionality constant. For the laser you will be using, the initial power of the laser beam is about $I_0 = 2$ milliWatts, which is about what you get from a weak laser pointer.

A brief word about laser safety. A good rule-of-thumb to remember is that staring directly at the sun puts about 1 mW into each of your eyes, so a 2 mW laser beam is enough to cause eye damage, at least in principle. But this only happens if you stare directly into the beam for at least a second or two, and this is all but impossible to do by accident. Thus a 2 mW visible laser (like a laser pointer) is quite safe to work with.

Now a key point in interferometry is that when the beams recombine at the beamsplitter, they do so by adding

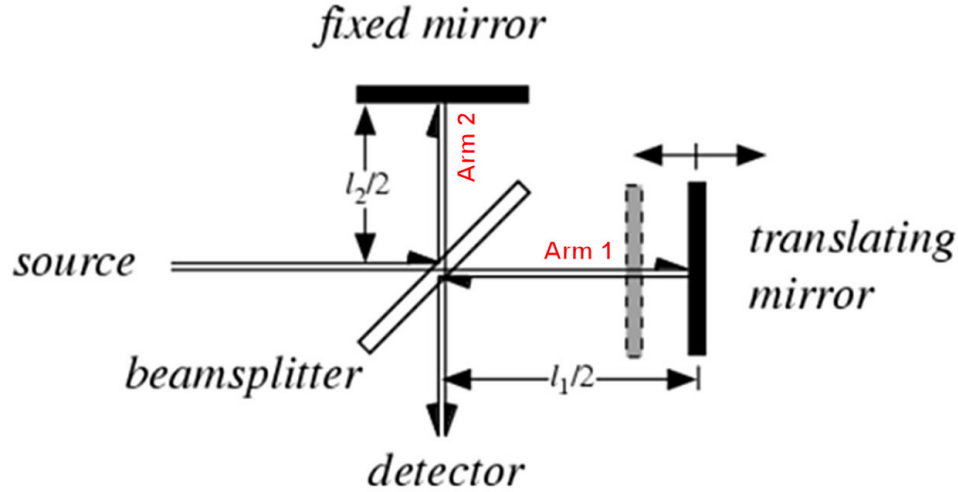


Figure 1. The optical layout of a basic Michelson interferometer. Light from the source (a He-Ne laser in our case) first strikes a 50:50 beamsplitter that splits the light into two equal-intensity beams. These beams travel down the arms of the interferometer, where they are then reflected back by two mirrors. The beams recombine back at the beamsplitter, so some of the light is directed toward the detector while some is reflected back toward the source. The intensities of these two beams depend on the path length difference $l_1 - l_2$ of the two arms of the interferometer. (Image from <http://scienceworld.wolfram.com/physics/FourierTransformSpectrometer.html>.)

their electric fields, and adding electric fields is not the same as adding powers. To see this, write the electric field of the recombined beam going toward the detector as (leaving off some of the extraneous factors for now)

$$E_{\text{det}}(l_1, l_2) \sim \left[\frac{E_0}{2} e^{-ikl_1} + \frac{E_0}{2} e^{-ikl_2} \right]$$

The first term in this expression represents the beam that traveled down the first “arm” of the interferometer (the beam path between the beamsplitter and a reflecting mirror in Figure 1) and back, a total distance l_1 , picking up the phase factor e^{-ikl_1} , while the second term represents the second beam that traveled down the second arm and back, a distance l_2 . Rewriting this

$$E_{\text{det}} \sim \frac{E_0}{2} [1 + e^{-ik\Delta l}] e^{-ikl_1}$$

where $\Delta l = l_2 - l_1$ is the path difference between the two arms, we then take the absolute square to get the power hitting the detector

$$\begin{aligned} I_{\text{det}} &\sim |E_{\text{det}}|^2 \\ &\sim \frac{E_0^2}{4} |1 + e^{-ik\Delta l}|^2 \\ &\sim \frac{E_0^2}{4} (1 + e^{-ik\Delta l}) (1 + e^{+ik\Delta l}) \\ &\sim \frac{E_0^2}{4} [1 + \cos(k\Delta l) - i \sin(k\Delta l)] [1 + \cos(k\Delta l) + i \sin(k\Delta l)] \\ &\sim \frac{E_0^2}{4} [2 + 2 \cos(k\Delta l)] \end{aligned}$$

If we do this carefully to get all the factors of two right, then the power hitting the detector becomes

$$I_{\text{det}} = \frac{I_0}{2} [1 + \cos(k\Delta l)] \quad (1)$$

We could do the same calculation for the light that ends up going back toward the detector (the other path through the beamsplitter; see Figure 1), and this would give

$$I_{\text{back to source}} = \frac{I_0}{2} [1 - \cos(k\Delta l)]$$

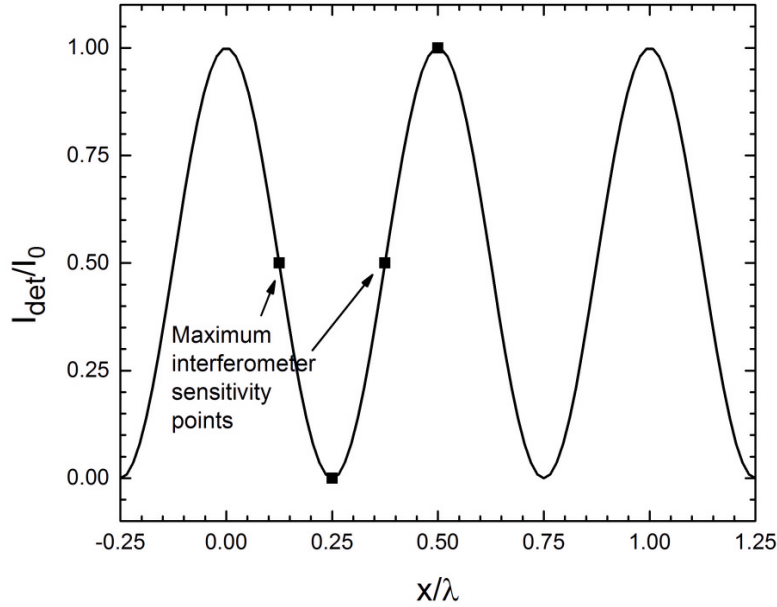


Figure 2. A plot of the power hitting the detector for an ideal interferometer, divided by the initial laser power I_0 , as a function of the displacement x of the translating mirror divided by the light wavelength λ . The interferometer is most sensitive to small changes in x at the points shown, when $I_{\text{det}} = I_0/2$.

Adding these two paths gives I_0 , which just means that energy is conserved.

Figure 2 shows a plot of I_{det}/I_0 as a function of x/λ for this ideal interferometer, where $x = \Delta l/2$ is the displacement of the translating mirror. (The path length change Δl is equal to $2x$ because the beam travels to the mirror and back again.) Note that I_{det} goes from a minimum of zero, when the cosine term equals minus one, to a maximum of I_0 when the cosine equals one.

Figure 2 is the most important plot in this experiment, so make sure you understand what it means. If not, ask someone about it.

Before you go into the lab and start playing with optics, let's take a closer look at how the interferometer shown in Figure 1 gets translated into a real experiment. We use a photodetector (a photodiode and some amplifying electronics) to convert the incident optical power to a voltage V_{det} that is proportional to I_{det} , converting the optical power I_{det} (in milliwatts) to a measured voltage V_{det} (in volts). In a real interferometer the two beams do not recombine perfectly, so the light intensity does not go all the way to zero as given in Equation 1. Putting all this together, the actual detector voltage becomes

$$V_{\text{det}} = \frac{1}{2}(V_{\text{max}} - V_{\text{min}}) [1 + \cos(2kx)] + V_{\text{min}} \quad (2)$$

where we have written this so that V_{det} goes from a minimum V_{min} to a maximum V_{max} .

If you want to measure large displacements with $x \gg \lambda$ using an interferometer, then you can do so by counting dark/light cycles as you translate the mirror. Our focus here will instead be on measuring very small displacements $x \ll \lambda$, and in the lab we will see just how small. In this case, the interferometer will be most sensitive to changes in mirror position x when dV_{det}/dx is maximum, as shown in Figure 2. Doing the math, we get

$$\begin{aligned} \frac{dV_{\text{det}}}{dx} &= \frac{1}{2}\Delta V (-2k) \sin(2kx) \\ &= -k\Delta V \sin(2kx) \end{aligned}$$

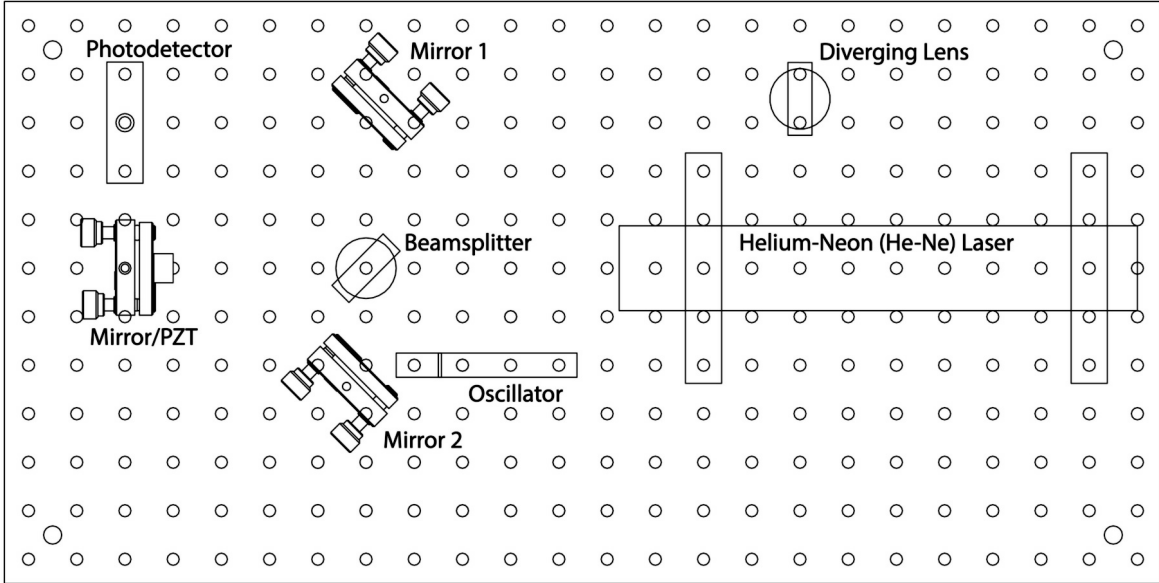


Figure 3. The optical breadboard (aluminum plate with holes) and the various optical elements. Not shown are two beam blockers (just paper cards on small stands) and the clamps that hold Mirror 1, Mirror 2, and the beamsplitter in place.

and

$$\begin{aligned} \left| \frac{dV_{\text{det}}}{dx} \right|_{\text{max}} &= k\Delta V \\ &= \frac{2\pi\Delta V}{\lambda} \end{aligned} \quad (3)$$

where we defined $\Delta V = V_{\text{max}} - V_{\text{min}}$.

Putting in $\lambda = 633 \text{ nm}$, we find that a displacement of the mirror by δx at this most sensitive point gives a change in V_{det}

$$\begin{aligned} \delta V_{\text{det}} &= \frac{2\pi\Delta V}{\lambda} \delta x \\ &\approx \frac{\delta x}{100 \text{ nm}} \Delta V \end{aligned} \quad (4)$$

Let's put this in terms of atoms, assuming an atom is about 0.1 nm in size. Then a mirror displacement of one atom gives an interferometer signal of $\delta V_{\text{det}} = \Delta V/1000$. In the lab you will measure that ΔV is about 7 volts. So if you can see δV down to 6 millivolts, then you can see a mirror displacement of just one atom (0.1 nm). If all goes well, you will be able to measure displacements substantially smaller than that.

2 Lab Procedures – Week One

Your main objectives for the first lab session are to: 1) build and align a “folded” Michelson interferometer, and 2) use it to examine the behavior of a mechanical oscillator in one arm of the interferometer. If you have already done the Magneto-Mechanical Harmonic Oscillator (MMHO) experiment, you will see some familiar concepts here (oscillating systems are common in physics). If not, this experiment will serve as an introduction to some oscillator physics, and you will see more when you do the MMHO experiment.

What follows are step-by-step instructions on how to accomplish these objectives. Each paragraph has a small task, and in series they will guide you through the lab. As you can see, there are a lot of steps, so you will need to work efficiently. If you find that one step is taking quite a while, you might want to ask for help before it chews up

too much time.

- First turn on the Laser Interferometry apparatus (the large on/off switch on the front panel) and the laser should go on; it may take a minute or two to warm up before the beam appears. While the laser is warming up, take a look at the optical elements on the breadboard. Figure 3 shows a diagram of how the previous group may have left things, with the various optical elements labeled. Note there are three optical elements held down with large, grey, fork-like clamps (Mirror 1, Mirror 2, and the Beamsplitter). Two of these pieces (Mirror 1 and Mirror 2), along with the Diverging Lens and the two beam blockers (white cards in stands), are yours to move around during this experiment.

The remaining pieces (the He-Ne Laser, the Photodetector, the Mirror/PZT, the Beamsplitter, and the Oscillator) are not meant to be moved, so **please do not try to move these pieces**. Also, the Mirror/PZT has been pre-aligned, so you should not need to align this mirror further. (More on what PZT stands for later.)

- Another important point: **please do not touch any optical surfaces**. The mirrors are front-surface mirrors, plus the beamsplitter and lens have dielectric coatings on both sides. All these surfaces are quite fragile and easy to damage. Fingerprints leave oils and acids that damage the coatings over time. If you see any fingerprints on any optical surfaces, or if the surfaces are dirty or damaged in any way, please tell someone. Do not try to clean the optics yourself; it is very easy to scratch the coatings while cleaning if you do not use the correct materials. Optical components are expensive, so please handle them with care.
- With all that in mind, begin by removing the two fork clamps holding Mirror1 and Mirror2. You can place the bolts and clamps in the tray provided. Be careful not to knock any of the pedestals over when you remove the clamps. Note how the clamps can hold down the optics at any position on the breadboard, and at any rotation angle. Set all these optic elements behind the laser (with the Diverging Lens in Figure 3) so they are out of the way. Use the white card provided to see the laser beam coming out of the laser, and note that the beam hits the Mirror/PZT and then reflects back toward the laser. On the front panel of the Laser Interferometry chassis, set the Lock/Scan switch to the Scan position, and remove any cables connected to the front panel.
- Next you should learn the five rules of aligning laser optics:
 - 1) Keep all laser beams at a fixed height. In your case, the optics are already set up to satisfy this rule.
 - 2) Keep the beams on a fixed right-angle grid. That is, make all beam reflections be right angles. This keeps things neat and orderly, and some optical elements (the 50:50 beamsplitter, for example) are designed for right-angle reflections.
 - 3) Make sure each beam hits each optical element close to its center. The optical elements are not large, and it is surprisingly easy to have a beam hit the mounting hardware instead of the intended optic. This is called *vignetting*, and you obviously don't want this to happen.
 - 4) Make especially sure that the beam hits the center of the photodiode. This is basically an extension of Rule 3, but it deserves special treatment because the photodiode is so small that there is little room for error.
 - 5) Keep all adjustment screws near the middle of their range. If an adjustment screw is near the end of its range, then soon it will lose its function. The adjustment screws are only needed for that last fine tweaking. So put the screws in the middle of their range and do the gross alignment before you tighten the clamps.
- With these rules in mind, your next task is to set up the optics as shown in Figure 4. We call this a “folded” interferometer because one of the arms is “folded” by Mirror 2. To begin, place a white card where Mirror 1 would sit, with the card perpendicular to the laser beam. You should see a spot on the card; note that the beam went through the beamsplitter, reflected off the Mirror/PZT, and then reflected off the Beamsplitter to make that spot.
- Next make sure both adjustment screws in Mirror 2 are near the center of their range – not all the way out or all the way in (Rule 5 above), and set Mirror 2 in place as shown in Figure 4 (but don't clamp it down yet). Align Mirror 2 so three things happen: 1) the incident laser beam hits Mirror 2 near its center (Rule 3), 2) the reflected beam hits the oscillator mirror near its center (Rule 3 again), and 3) the beam is “retroreflected” off the oscillator – that is, the beam reflecting off the oscillator mirror goes in the opposite direction of the incident beam. Note that the retroreflected beam will then appear as a second spot on the card, near the first spot.

It is not trivial to make all these things happen at once, since you only have three degrees of freedom to adjust: the x- and y- positions of Mirror 2, and the horizontal pointing of Mirror 2. To make the alignment work,

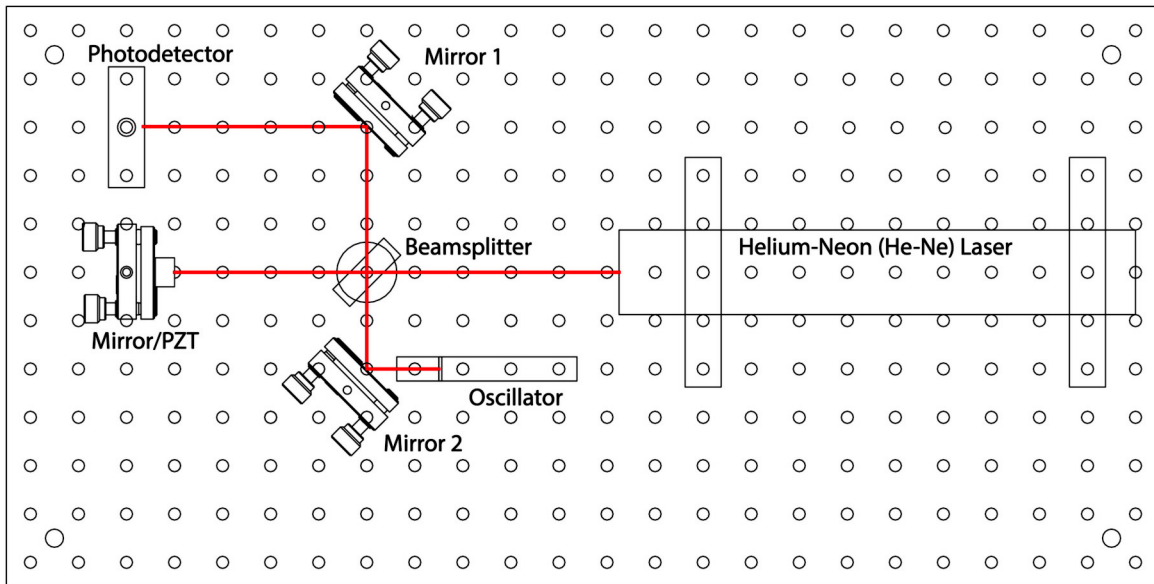


Figure 4. The interferometer set-up for the second week. Here Mirror 2 is a folding mirror, while the small mirror on the oscillator retroreflects the beam.

you must use all these degrees of freedom correctly, so the alignment becomes something of a geometrical puzzle. Once Mirror 2 is about where you want it, clamp it down using the fork.

You may find that the fork clamps take a bit of getting use to also. A good strategy is to first place Mirror 2 about where you want it and decide which hole you will use for the clamp—one that is close to the base of the optical mount. Then place the clamp in loosely while not worrying about the mirror alignment. Finally align the mirror about how you want it and tighten the clamp. Use the Allen wrench to tighten the bolts pretty tight, but no more than what you can do easily by hand. No need to be a gorilla.

- The final tweaking of the alignment is then done with the alignment screws on the Mirror 2 mount (do not adjust the Mirror/PZT alignment). Align Mirror 2 so the two bright beams (from Mirror 2 and the Mirror/PZT) overlap on the card. They should be right on top of one another. (You may see some fainter beams also on the white card; these are additional reflections from the beamsplitter, and they can be ignored.)
- Now place the Diverging Lens directly in front of the laser and observe what's on the card. You may have to move the lens around a bit to keep the beam, now quite large, from vignetting on the different optical mounts. Once you have a nice round spot on the card, you should be able to tweak the alignment of Mirror 2 to see a bull's-eye pattern of *fringes* on the white surface. Observe how the pattern changes as you tweak the Mirror 2 alignment. (If you see no fringes, remove the lens and point Mirror 2 again so the two beams overlap on the card).

To see the fringes really well, set Mirror 1 down so it reflects the beam to the right (not toward the Photodetector, but in the opposite direction), and place the card in the upper right corner of the breadboard. You do not have to clamp Mirror 1 down for this; just set it in place. The extra fold just lets the diverging beam travel farther, so it diverges into a larger spot. Then place the diverging lens right after the beamsplitter, between it and Mirror 1. Align Mirror 2 so the fringe pattern turns into one large fringe—this means the interferometer is properly aligned.

- Note that if you push gently down on the aluminum breadboard, you can see the fringes move. This is because you are flexing the metal plate, which in turn changes the lengths of the interferometer arms. That's a half-inch-thick plate, so you are not bending it very much. But half a wavelength is a pretty small distance, so it doesn't take much of a push to see the fringes go through one cycle. If all is aligned well, then pushing down on the breadboard will cause the laser spot to go from bright to dark. The spot will not become completely dark, however, because it is nearly impossible to achieve a perfect alignment (plus there are always some imperfections in the optics). If you try it, you will see that you cannot achieve a perfect null on the white card.
- Once you understand the interferometer fringes to your satisfaction, remove the Diverging Lens and set up the interferometer shown in Figure 4, this time with Mirror 1 clamped down. Check that all the clamps are tight, and

that the two main beams overlap at the photodiode, and are centered on the photodiode. Note how there are again just enough degrees of freedom to produce an aligned interferometer. The Mirror 2 alignment sets the overlap of the two arm beams; and Mirror 1 sends the beam to the center of the photodiode.

- Once the optical setup looks good, connect the Signal Generator OUT-1 to channel 1 of the oscilloscope. Trigger the oscilloscope on channel 1, and you should see a square wave or a triangle wave.
- The knobs and switches on the right-hand side of the front panel are there to control a simple signal generator inside the main chassis. There is a switch to change between a triangle-wave or square-wave output, and knobs to control the signal generator amplitude and frequency. Flip the switch and turn the knobs to see what happens to the signal generator output on the oscilloscope. Then turn the Frequency Coarse Adjust knob all the way down, turn the Amplitude Adjust knob all the way up, and set the switch so you see a triangle wave on the oscilloscope.
- Next connect the Signal Generator OUT-2 (which is internally connected to OUT-1) to the Scan IN, and verify that the Lock/Scan switch is set to Scan. Finally, connect the Photodiode OUT signal to channel 2 on the oscilloscope, and set things up so you see both signals.
- What's happening is this: By sending the triangle wave to the Scan IN port, you are sending a voltage to a piezoelectric transducer that sits behind the small mirror in the Mirror/PZT housing. (PZT stands for PieZoelectric Transducer; or for Lead(Pb)-Zirconate-Titanate, the most commonly used piezoelectric material.) The PZT expands when a voltage is applied, so the mirror position x is proportional to the voltage you apply. Put another way, the signal you see on the oscilloscope channel 1 is proportional to the mirror position x . The triangle wave means that the mirror is simply moving back and forth in a linear fashion. Meanwhile, the Photodiode OUT signal gives a voltage proportional to the light power hitting the photodiode. By observing these two signals together on the oscilloscope, you can see the interferometer signal $V_{\text{det}}(x)$ as given in Equation 2, and shown in Figure 2. You should see an oscillatory signal, but one that goes back and forth as x executes a triangle wave. With the Amplitude Adjust turned all the way up, you should see 2-3 fringes on the oscilloscope. If you are not seeing this, check the optical alignment or ask someone for help. Push down gently on the breadboard and watch how the fringes move on the oscilloscope.

Make sure you really understand what you see on the oscilloscope at this point (if not, ask). If you put the oscilloscope in XY mode, you can see $V_{\text{det}}(x)$ more directly (because V_{det} is on channel 2 (Y), while the PZT voltage (proportional to x) is on channel 1 (X)).

- With the interferometer signal on the oscilloscope, you can use this new information to better optimize the optical alignment. First, tweak the Mirror 1 pointing to maximize V_{max} ; this makes sure the beam is hitting the center of the photodetector. Second, tweak the Mirror 2 pointing to minimize V_{min} ; this makes sure the interfering beams overlap as much as possible. You can turn up the gain on the oscilloscope to see V_{min} better while adjusting Mirror 2. Repeat these steps until you optimize the alignment. You should be able to get to $V_{\text{min}} < 0.5$ volts, and V_{max} should be around 7 volts or more. At this point, you can stop and pat yourself on the back for putting together a well-aligned laser interferometer.
- Just in passing, you should be aware that the piezoelectric effect is a wonderfully useful method for moving things around electronically. It is used to focus lenses in cameras, to position things in scanning probe microscopes, to actuate quartz crystal oscillators in clocks, and in a host of other applications. The study of piezoelectric materials is a substantial area of current research, and new applications are being found all the time. If you ever need to position small objects carefully over small distances, you should remember piezoelectrics.
- So, now that you have a working laser interferometer, you can use it to do what interferometers do best – measuring small motions. To proceed, the next step is to “lock” the interferometer at its most sensitive point, and this is done electronically by monitoring $V_{\text{det}}(x)$ and then setting the PZT so V_{det} equals a set voltage V_{set} . We do not have time to discuss all the details of how this happens, but you can see it in action by setting the Set Point Adjust knob to the center of its range and flipping the Scan/Lock switch to the Lock position. The LED near the switch should stay green.
- When in Lock mode, the interferometer output on the oscilloscope should become a nearly constant value (jumping around some as the interferometer is being jostled). The electronics tries to keep $V_{\text{det}} = V_{\text{set}}$, with V_{set} determined by the Set Point Adjust knob. If you turn this knob up and down (slightly), the interferometer output should follow. Use the oscilloscope output in the Scan and Lock modes to adjust the Set Point Adjust so

$V_{set} = V_{min} + (V_{max} - V_{min})/2$ – that is, so the lock point is halfway between V_{min} and V_{max} . When this is done, then dV_{det}/dx is maximized (consider Figure 2 to see why this is true). By setting x to maximize dV_{det}/dx , it will then be easiest to see small changes in x .

- The electronic circuit behind all this is called a *feedback loop*, as the measured V_{det} is “fed back” to change x , which in turn changes $V_{det}(x)$. This is also called a *servo mechanism*, and the math that describes it is part of *Control Theory*, which is a branch of engineering. Servo mechanisms are used in your home (for example in the thermostat controlling the room temperature), in your car, in your electronic devices, and of course in practically every modern physics experiment. Whenever you need to control something – a temperature, a current, a voltage, a position, etc. – you will likely do so using a servo mechanism. You can take whole courses describing Control Theory in detail if you are interested (see the ME section of the course catalog).
- Notice the LED next to the Scan/Lock switch. If the interferometer is in Lock mode and this LED is green, then all is well; the lock should be working. But if you tap on the breadboard a bit, the servo may eventually stop working and the LED will turn red. The reason is that if x_{lock} satisfies $V_{det}(x_{lock}) = V_{set}$, then $x_{lock} + \lambda/2$ will satisfy it also (again consider Figure 2 to see why this is true). The servo will pick the nearest x_{lock} when you turn it on, but it may later jump to a different x_{lock} requiring a larger or smaller PZT voltage. If this voltage gets too high or too low, then the servo can no longer function properly. The bottom line is that if you see a red LED, then the interferometer is no longer properly locked. If the LED turns red, switch the lock off and on again to reset it. If this does not turn the red light to green, ask for help.
- If you take a closer look at the oscillator (see Figure 4), you will see what is basically a rectangular aluminum post with a small mirror attached to one side and a magnet attached to the other side. The post behaves like a rigid body that pivots about the fixed attachment point at the base of the post (like one arm of a tuning fork). In a nutshell, this device behaves like a simple harmonic oscillator; if moving freely, the post oscillates back and forth with some resonance frequency ν_0 . If you send an oscillating current through the nearby coil, the resulting magnetic field pushes and pulls on the magnet, and you have a driven harmonic oscillator.
- Next drive the oscillator using the small drive coil on the oscillator mount. Connect Signal Generator OUT-2 to Coil Drive IN, which sends a current through the coil, creating a magnetic field that pushes/pulls on the oscillator magnet. Verify that OUT-1 is connected to channel 1 of the oscilloscope, and that the Photodiode OUT signal is connected to channel 2 of the oscilloscope.
- With the interferometer locked, you can then drive the mirror back and forth using the coil/magnet (the triangle wave signal on channel 1), while you watch the mirror oscillating (the photodiode signal on channel 2). When you first plug this in, it is likely that you are not driving the oscillator on resonance, so you will not yet see much on channel 2 of the oscilloscope. Measure the channel 1 frequency (which has a strong signal) using the measure feature on the oscilloscope.
- Now adjust the signal generator frequency while watching the oscilloscope. When you are very close to the resonance frequency of the oscillator (around 700 Hz), you should see the oscillator respond. The response will peak sharply when you are right at the resonance frequency. Use the Frequency Fine Adjust knob to maximize the oscillator response, and set the Amplitude Adjust knob so the peak-to-peak response of the interferometer is about $\Delta V/3$, about 2 volts. With this as the peak response, the interferometer response will still be in the linear regime, so the voltage modulation will be proportional to the mirror position modulation.
- Maximize the oscillator response and use the oscilloscope to measure the drive frequency (of channel 1 on the oscilloscope). You should be able to measure ν_0 to about 5 Hz accuracy this way. Try to measure ν_0 carefully, and write the value down in your notebook. You may find it useful to use the average feature on the oscilloscope to average 4 or 16 traces, giving a cleaner signal.

Now we need to step back from the experiment and look briefly at the harmonic oscillator math. Since the post pivots about its base, this system is best described as a torsional harmonic oscillator. If you displace the angle of the post by a small amount θ from its equilibrium position, then this flexes the aluminum at the base and results in a restoring torque $\tau = -\kappa\theta$. The equation of motion in angular coordinates becomes $\tau = I\ddot{\theta}$, where I is the mass moment of inertia.

Although these are the best equations for describing the oscillator motion, the interferometer is not very sensitive



Figure 5. A series of weights you can add to the oscillator, here arranged from lightest (on the left) to heaviest. You should find a pair of each of these in the tray.

to θ , but instead measures the mirror position x . So it is convenient to change variables to $x = R_{post}\theta$, where R_{post} is the length of the aluminum post. Doing this gives us the new equations of motion

$$F = m_{eff}\ddot{x}$$

and

$$F = -k_{eff}x$$

where $m_{eff} = I/R_{post}^2$ can be thought of as an effective mass of the oscillator, and $k_{eff} = \kappa/R_{post}^2$ is an effective spring constant. (You can easily do all the math to verify these relationships; all we did here was a change of variables.)

Putting these two equations together gives the harmonic oscillator equation

$$m_{eff}\ddot{x} + k_{eff}x = 0$$

and solving this gives oscillatory solutions with the resonant frequency

$$\omega_0 = \sqrt{\frac{k_{eff}}{m_{eff}}}$$

where $\omega_0 = 2\pi\nu_0$ as usual (ν_0 is the oscillation frequency in Hz, or cycles/second, while ω_0 is the angular frequency in radians/second).

Now that we have the mathematics we need, we proceed with more step-by-step instructions:

- Your next goal is to change the mass of the oscillator and see how this changes the resonance frequency. Figure 5 shows a series of weights you should use for this purpose; you should have a pair of each of these in your tray. The weights screw into the holes near the top of the oscillator post, and for balance you should put identical weights on either side of the post. Turn off the servo when you change weights, and of course do not touch the mirror surface at any time. Screw the weights in snugly, but only with a modest two-finger-only tightening; again, no gorilla hands.
- Weigh each pair of weights before or after you measure the new resonance frequency. You should have a small electronic scale for this (a black box labeled AWS). Please be gentle with this scale, as it is a precision instrument. Set it down gently on the table, and set the weights down gently when you weigh them. If you blow down onto the scale, you can see how sensitive it is. The scale reads to 0.01 grams, but the absolute accuracy is probably closer to 0.03 grams. If you weigh one mass, then the other, and then both together, you can get some idea about the actual accuracy.

- Go through all seven sets of weights, recording the mass of each pair m_{added} and the resonance frequency of the oscillator with the weights attached. Plus you have your first point with $m_{added} = 0$. Plot all eight points; specifically, plot ν_0 as a function of m_{added} in a Mathematica notebook. If you are not yet very familiar with Mathematica, and you are unsure how to proceed, consult the example notebook in the Appendix below.
- You expect your data to be described by

$$2\pi\nu_0 = \sqrt{\frac{k_{eff}}{m_{eff} + m_{added}}} \quad (5)$$

so a fit to the data would give you both k_{eff} and m_{eff} . In this form, however, the equation would require a nonlinear fit. While this is possible, a linear fit is quicker and easier. This can be accomplished by rewriting the above as

$$\begin{aligned} 4\pi^2\nu_0^2 &= \frac{k_{eff}}{m_{eff} + m_{added}} \\ \frac{1}{4\pi^2\nu_0^2} &= \left(\frac{1}{k_{eff}}\right)m_{added} + \frac{m_{eff}}{k_{eff}} \\ &= Am_{added} + B \end{aligned}$$

With this reformulation in mind, plot $1/4\pi^2\nu_0^2$ as a function of m_{added} in your Mathematica notebook. This time you should get a straight line. Apply a linear fit to the data, and use the fit slope and intercept to determine m_{eff} and k_{eff} , and write both quantities in your notebook. (Make sure the numbers have units.)

Now a measurement is not complete without an error estimate. For standard 1-sigma errors, here is a handy shortcut. We can write the following:

$$\begin{aligned} k_{eff} &= A^{-1} \\ \frac{dk_{eff}}{k_{eff}} &= -\frac{dA}{A} \end{aligned}$$

where $A = k_{eff}^{-1}$ is your slope fit parameter, and the second line is just the derivative of the first line. This relationship means that the error bars propagate as (see Taylor)

$$\begin{aligned} \frac{\sigma_{k_{eff}}}{k_{eff}} &= \frac{\sigma_A}{A} \\ \sigma_{k_{eff}} &= \frac{\sigma_A}{A^2} \end{aligned}$$

which gives you $\sigma_{k_{eff}}$ once you have A and σ_A . Similarly, for the other term

$$\begin{aligned} m_{eff} &= Bk_{eff} \\ \frac{dm_{eff}}{m_{eff}} &= \frac{dB}{B} + \frac{dk_{eff}}{k_{eff}} \\ \frac{\sigma_{m_{eff}}}{m_{eff}} &= \frac{\sigma_B}{B} + \frac{\sigma_{k_{eff}}}{k_{eff}} \end{aligned}$$

where $B = m_{eff}k_{eff}^{-1}$ is the constant fit parameter. You can use this shortcut whenever you have power-law relationships between variables. It has the nice feature of delivering σ_X/X directly, which is the fractional error in a measured quantity.

3 Lab Procedures – Week Two

Your task this week is to examine different methods for measuring really small displacements with this interferometer, and to see just how low you can go. With a bit of effort, you should be able to detect displacements down to about one picometer, which is 1/100th of an atom. As before, we have some step-by-step instructions to guide you through it.

- Begin in the lab by setting up the interferometer as shown in Figure 6. The set-up and alignment procedure is

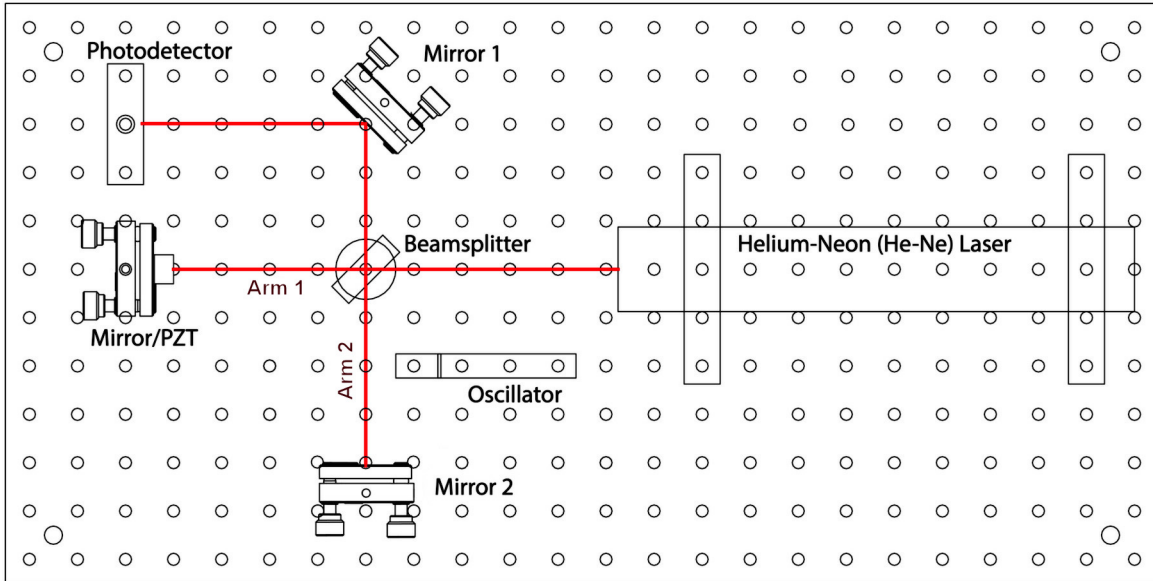


Figure 6. The optical setup for a basic Michelson interferometer. Note both mirrors are about 10 cm (4 inches) from the beamsplitter, and note how the beams follow right angles defined by the hole pattern.

the same as you did last week (easier without the folding mirror), so we do not repeat it here. However, this time place Mirror 2 so the two interferometer arms have very nearly the same length. Use a plastic ruler to measure the arm lengths to make them identical to a couple millimeters. Make sure both Mirror 1 and Mirror 2 are clamped down well.

- As you did last week, center the combined beam on the photodiode, connect the Signal Generator OUT-1 to channel 1 of the oscilloscope, and connect the Photodiode OUT signal to channel 2 on the oscilloscope, and set things up so you see both signals. Trigger the oscilloscope on channel 1, and you should see a square wave or a triangle wave, as you did last week. Adjust the oscillator settings to give a triangle wave set to the lowest frequency and highest amplitude. Connect the Signal Generator OUT-2 to the Scan IN, and verify that the Lock/Scan switch is set to Scan. (Remember that the Signal Generator OUT-1 and OUT-2 ports are identical.) Now you should see the interferometer fringes on the oscilloscope, same as you did last week. If this is not working, you probably don't have the interferometer properly aligned, so take a look again at the instructions from last week.

- When you have the optics nicely aligned and the interferometer signal on the oscilloscope, tweak the Mirror 1 pointing to maximize V_{\max} , and tweak the Mirror 2 pointing to minimize V_{\min} , as you did last week. You should be able to get to $V_{\min} < 0.5$ volts, and < 0.2 volts if the alignment is good. V_{\max} should be around 7 volts or more.

- When you have minimized V_{\min} , measure the fringe *visibility*, sometimes called the fringe *contrast*, defined by

$$S = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

with possible values ranging from 0 to 1. The Measure feature on the oscilloscope is useful here, since it will show you V_{\min} and V_{\max} directly. You should be able to get $S > 0.9$. Write this value in your notebook, and do a screen shot of the oscilloscope (just press the print button if you have a networked oscilloscope, or photograph the screen) and tape this into your notebook. Record V_{\min} , V_{\max} , and $\Delta V = V_{\max} - V_{\min}$ as well.

- From the signal you see on the oscilloscope, calculate dx/dV_{PZT} . That is, calculate how much the length of the PZT changes when you change the applied voltage. Everything you need for this is right there already on your oscilloscope. For example, the voltage V_{PZT} is what you are sending to the Scan IN port, therefore V_{PZT} is also on channel 1 of the oscilloscope. Meanwhile, the change in x can be seen from the interferometer response, and you already know $\lambda = 633$ nm. Remember that one fringe (bright to bright) means $dx = \lambda/2$ (why? See Figure

2). To measure dx/dV_{PZT} , freeze a good trace on the oscilloscope (by pressing the Run/Stop button), and then use the vertical cursors on the oscilloscope to measure how much V_{PZT} changes (i.e., dV_{PZT}) when $dx = \lambda/2$. Compare your answer with the manufacturer's specifications for this PZT, which say it has a displacement of $10 \pm 1 \mu\text{m}$ at 150 volts, and record all this in your notebook.

- Once the interferometer is set up and aligned, lock it so $V_{set} \approx (V_{max} + V_{min})/2$, same as you did last week. Note that if you press gently on the breadboard when the interferometer is scanning, you will see the fringes move on the oscilloscope. But if you press down slowly on the breadboard when the interferometer is locked, the output stays at V_{set} . This is because the PZT compensates when you change the interferometer arm lengths. The PZT continually changes the Mirror/PZT position to keep V_{det} equal to V_{set} .
- Now the fun begins; the next step is to modulate the mirror position x while the interferometer is locked. To see this happen, lock the interferometer, turn the Amplitude Adjust knob all the way down, and connect Signal Generator OUT-2 to Modulation IN (disconnecting from Scan IN). Leave channel 2 of the oscilloscope connected to Photodiode OUT, and connect channel 1 to Sync OUT. Set the Multiplier knob to 100, switch the signal generator over to a square wave, and turn the Servo Gain Adjust all the way down. Finally, turn up the Amplitude knob a bit until you see a small square wave modulation on V_{det} . That is, V_{det} will equal V_{set} plus a small square wave modulation.

Two things are happening here. First, the servo is in Lock mode, so on average V_{det} should be equal to V_{set} . But in addition to this, sending the signal generator output to Modulation IN tells the PZT to add a modulation given by the signal generator. The two commands to the PZT are separated in frequency: at low frequencies the servo wins, keeping $V_{det} = V_{set}$ on average. But at high frequencies the signal generator wins, causing the PZT position to be modulated by the input square wave.

- Sometimes these two controls work against one another. To see this, turn the square wave frequency all the way down, then adjust the Servo Gain and see what happens. When the servo gain is low, then the servo only operates at very low frequencies, so the square wave modulation is seen faithfully on V_{det} . But when the servo gain is high, then the servo works harder to keep $V_{det} = V_{set}$. When the square wave modulation causes a jump in V_{det} , the servo sees this and adjusts x to compensate, bringing V_{det} back down to V_{set} within one half-cycle of the square wave. As a result, the square wave in V_{det} becomes distorted. There is quite a bit going on at once here, so it can be difficult to understand at first. But once you see it, it is not so complicated and it actually makes sense. Ask someone at this point if you need help absorbing it all.
- Next turn the signal generator frequency to about 700 Hz (use the Measure feature on the oscilloscope, looking at channel 1), and turn the Servo Gain to its maximum. Use AC coupling for channel 2 on the oscilloscope and turn up the gain knob on the oscilloscope. Turn the Amplitude Adjust knob so the square wave you see in V_{det} has a top-to-bottom amplitude of about 1 V. If you turn the modulation frequency down, you will see the distortion that happens when the servo partially compensates for the modulation. If you turn the frequency up above 700 Hz slowly, you will see some mechanical resonances in the mirror housing. Shaking the mirror at these high frequencies causes the mirror housing to shake as well, and the amplitude of this motion peaks when you hit a mechanical resonance.
- Adjust the signal generator frequency so that the V_{det} signal (channel 2 on the oscilloscope) looks similar to the Sync signal (channel 1 on the oscilloscope). The Sync signal has been rounded a bit (by sending it through a low-pass filter) to mimic the V_{det} response that has been similarly filtered. So you should be able to find a frequency where the two signals both look like similarly rounded square waves, somewhere in the neighborhood of 700 Hz.
- Now verify that the Multiplier knob is set to $M_0 = 100$, and adjust the Amplitude knob so that the top-to-bottom amplitude is equal to $\Delta V/10$, which will be about 700 mV. Average 64 traces to see a cleaner signal (under the Acquire menu), and use the measure feature on the oscilloscope to measure the peak-to-peak voltage. Since $\Delta V/1000$ corresponds to a mirror shift of one atom (see Equation 4), we see that $\Delta V/10$ corresponds to 100 atoms, or 10 nm. Although 10 nm is a pretty small distance (about 5,000 times smaller than the diameter of a human hair!), it is easily seen with the interferometer. The signal you see on the oscilloscope should have a high signal-to-noise and be easy to measure. Print out a screen shot and add it to your notebook. Again, if this is not making sense, ask someone for help.
- From this point on, do not move the Amplitude knob. Then the mirror's back-and-forth displacement Δx will be proportional to M_0 . From the way you set the Amplitude knob (previous paragraph), you know that Δx is about

equal to M_0 atoms, or equivalently $\Delta x \approx (M_0/10)$ nm.

- Next turn the Multiplier knob down to 10, which then corresponds to a mirror displacement of just 10 atoms ($\Delta x = 1$ nm). If you increase the gain on the oscilloscope accordingly, you will now see that V_{det} is quite a bit jumpier. The square wave amplitude should be down to about $\Delta V/100$, or 70 mV. The noise mostly comes from seismic noise and acoustic noise. You could reduce the seismic noise by adding some additional rubber pads under the interferometer chassis, but just not bumping the chassis or the table helps quite a bit. (Note that the seismic noise here has little to do with seismic waves in the Earth; it is mostly from human-caused shaking of the floor and table.) Acoustic noise shakes the optical mounts directly, and you can see this on the oscilloscope when you talk or clap your hands. Keeping quiet helps reduce this noise, of course.
- Now use the average feature of the oscilloscope to average 128 (or more) traces. This slows down the response time, but substantially reduces the jitter. This works because the modulation signal is tied to the Sync signal, which the oscilloscope is triggering on, while all the noise is random in time. Thus averaging traces reduces the noise but not the signal. With some averaging, the modulation signal should be quite clear even with only 10 atoms of mirror motion.
- Note how you have used several methods to enhance the sensitivity of the interferometer:
 - 1) First, you are modulating the mirror position at high frequencies. This helps because most of the noise, especially the seismic noise, is at low frequencies. This is a fairly ubiquitous phenomenon, broadly called *1/f noise*, since for many noise sources the noise intensity scales roughly as one-over-frequency. Putting the modulation signal at high frequency (700 Hz) means lower noise. If you go back to sample mode on the oscilloscope (not averaging), you can see that most of the noise is just the trace jumping up and down. This is low-frequency noise, and it averages away nicely when you switch back to average mode on the oscilloscope.
 - 2) Second, you are modulating the mirror position in a known way, in this case a square wave modulation. Because you then know how the V_{det} signal should be responding, you can look for just this response in the signal.
 - 3) And finally, you are averaging data. For random noise, the noise amplitude will be reduced by $1/\sqrt{N}$, where N is the number of averages.
- So far, so good. Next turn the Multiplier knob to 1, meaning a modulation of just one atom, or 0.1 nm. Keep the averaging on, and increase the gain on the oscilloscope to 20 mV/division. There is a lot of noise now, but you should still be able to see the square-wave signal in the V_{det} trace on the oscilloscope, and the peak-to-peak modulation should be roughly $\Delta V/1000$ or 7 mV.
- We have one more trick up our sleeve, and that is to increase N still further. At 700 Hz, one cycle of the square wave takes a bit over 1 millisecond, so in 10 seconds you can average nearly 10,000 cycles, so then \sqrt{N} gives a factor of 100 – not a small gain in signal-to-noise. This averaging has been done for you electronically in the interferometer using something called *phase-sensitive averaging*. In a nutshell, the electronics does a running average of the photodiode signal when it knows it should be high (the high parts of the square wave), and it does another average when it knows the signal should be low. Then it differences those two running averages, sending the difference signal (after some additional amplification) to the Amplitude OUT port on the front panel (call this signal V_{diff}). Thus V_{diff} will be a DC signal that is proportional to Δx . To see this for yourself, turn the Multiplier knob to 10, and view the oscillating signal on the oscilloscope while you measure V_{diff} using a digital voltmeter. If $V_{\text{diff}} = A\Delta x$, what is A (in volts per nanometer)? Record A in your notebook.
- Next turn the Multiplier knob to 0, which means there is no modulation being sent to the PZT. In principle, V_{diff} should go to zero, but the signal-averaging is not done perfectly in the electronics, so you will see a non-zero V_{diff} . Record V_{diff} every five seconds and write these numbers down. Once you have 10-20 numbers, calculate the means and standard deviation. This gives you $V_{\text{diff}}(0)$, the signal with no modulation. This is just a simple offset voltage, so subtracting it gives $V_{\text{diff,corrected}} = V_{\text{diff}} - V_{\text{diff}}(0)$, and this quantity does go to zero when there is zero modulation going to the PZT.
- Finally set the Multiplier knob to $M = 1$, and to 0.1, and measure $V_{\text{diff,corrected}} = V_{\text{diff}} - V_{\text{diff}}(0)$ for both, with error bars. On a good day, when the external noise sources are not bad, both the $M = 1$ and $M = 0.1$ measurements should give you 1-sigma errors of about 0.01 atoms, or one picometer. Record your best 1-sigma measurement error in picometers.

- Note that your 1-sigma error is for measuring changes of $x = dL$ in the length of the interferometer arm L , which is about 10 cm. So what is the dimensionless dL/L , called the *strain sensitivity* of the interferometer, if dL is 0.01 atoms? If you wanted to measure the distance from New York to Los Angeles (4500 km) with the same strain sensitivity, about what dL would you need?
- So, what should you take away from this experiment? First of all, you now know a bit about setting up a laser interferometer. You have seen the interferometer fringes with your own eyes, and you have seen the detector signal. You are now aware that piezoelectric devices can be used to push things around. And you have seen that laser interferometry can be used to measure very small displacements, and that there are a number of tricks for improving the measurement sensitivity.
- People at Caltech and elsewhere have worked for many decades to push laser interferometry to ever greater sensitivities, by carefully reducing seismic noise (by building vibration isolating mirror suspensions) and acoustic noise (by putting the interferometer in a vacuum), until finally one is dealing with quite fundamental noise sources, for example shot noise (the fact that one is making a measurement using a finite number of photons) and even radiation pressure from the laser beams themselves. The strain sensitivity of the LIGO (Laser Interferometer Gravity-wave Observatory) interferometers is currently below 10^{-22} for $L = 4$ kilometers. That's like measuring the distance to the nearest star (besides the Sun) to the width of a human hair!

Fitting Interferometer data, week 1: Frequency vs. Added Mass

For this measurement, you recorded the frequency of the blue oscillator as a function of the mass you added (including the zero-added-mass point).

Now you want to fit those data to extract the m_{eff} and k_{eff} , as described in the handout.

Your first step is to enter your numbers into a *Mathematica* list, for example (only showing two points here):

```
data1 = {{mass1, freq1}, {mass2, freq2}}
{{mass1, freq1}, {mass2, freq2}}
```

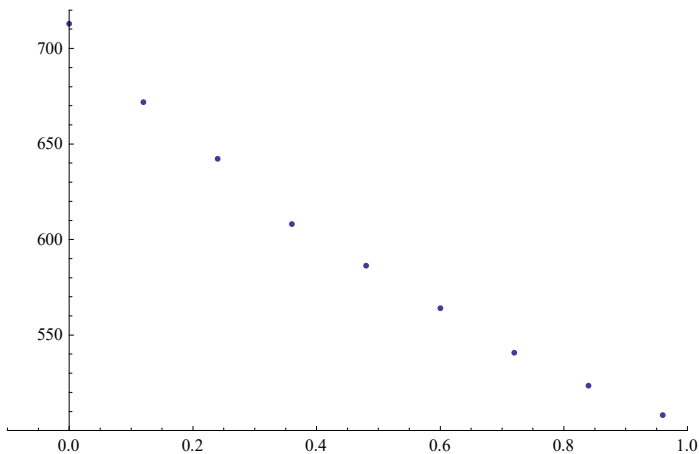
For this demonstration, we will use some simulated data (with arbitrary units here):

```
meff = 1; (* the semicolon suppresses output for this line *)
keff = 2 * 10^7;
data = Table[{madd, Sqrt[keff / (meff + madd)] / (2 * Pi) +
  2 * RandomVariate[NormalDistribution[]]}, {madd, 0, 1, .12}]
{{0., 712.986}, {0.12, 671.868}, {0.24, 642.226}, {0.36, 608.133}, {0.48, 586.337},
  {0.6, 564.025}, {0.72, 540.762}, {0.84, 523.551}, {0.96, 508.117}}
```

As you can see, we used Equation 5 in the handout to generate our simulated data, along with some added random noise.

Next plot your data set to check it. You may be tempted to skip this step, but don't; if you do not check your work as you go (in this case, your data entry), errors often creep in. This plots the points:

```
ListPlot[data, PlotRange → {{-0.1, 1}, {500, 720}}]
```



As described in the handout, $\text{freq}(\text{madd})$ is not a linear function. You could use `NonlinearModelFit` to fit the data, but nonlinear fits can be tricky. Better to convert the data to make a linear function, as described in the handout.

So we extract the frequency data by itself and make a new list:

```

freq = data[[All, 2]];
mass = data[[All, 1]];
data2 = Table[{mass[[i]], 1 / (4 * Pi^2 * freq[[i]]^2)}, {i, 1, Length[mass]}]

```

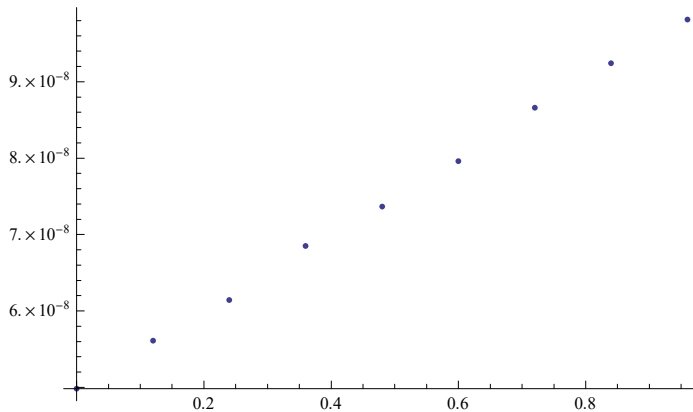
```

{{0., 4.98285 × 10-8}, {0.12, 5.61142 × 10-8}, {0.24, 6.14136 × 10-8},
 {0.36, 6.84926 × 10-8}, {0.48, 7.36793 × 10-8}, {0.6, 7.96239 × 10-8},
 {0.72, 8.6622 × 10-8}, {0.84, 9.24107 × 10-8}, {0.96, 9.811 × 10-8}}

```

As promised, plotting this shows a simpler linear relationship:

```
ListPlot[data2, PlotRange → All]
```



Then do a linear fit to the data (no parameter guesses are needed for linear fits; they pretty much always work as expected):

```
fitline = LinearModelFit[data2, {1, x}, x]
```

```
FittedModel [ 4.97952 × 10-8 + 5.04949 × 10-8 x ]
```

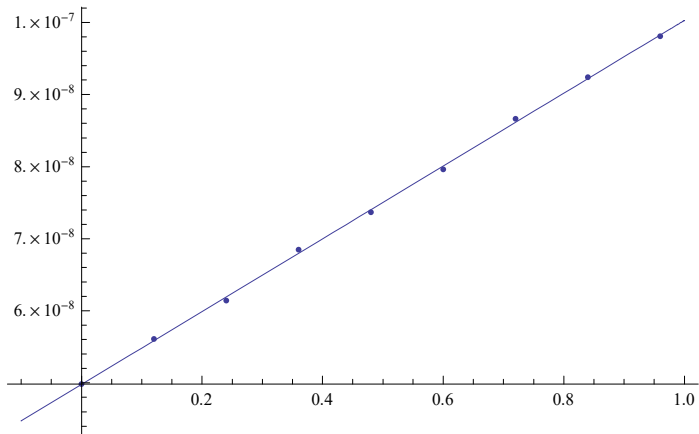
And print out the parameters with standard errors :

```
fitline["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
1	4.97952 × 10 ⁻⁸	2.56554 × 10 ⁻¹⁰	194.092	2.54373 × 10 ⁻¹⁴
x	5.04949 × 10 ⁻⁸	4.4906 × 10 ⁻¹⁰	112.446	1.15991 × 10 ⁻¹²

And plot the fit with the data points:


```
Show[ListPlot[data2, PlotRange → All], Plot[fitline[x], {x, -0.1, 1}]]
```



And finally, you can use the fit parameters to extract k_{eff} and m_{eff} , as described in the handout.