Ge/Ay 117 Example Problem Set and Solutions
Spring 2014

In this course, the problems in your sets will generally be asking you to perform a calculation, write code to do a computation (and explain it), and to plot things in various formats. This example problem set includes all these types and my sample solutions contain an example of what I would be expecting when I grade your problem sets.

1) Suppose you wanted to generate a probability density function, pdf(x), where only values between 10 and 100 are allowed and you want pdf(x) to be proportional to 1/x (i.e. strongly favour lower values of x). Therefore, you can write

$$pdf(x) = A / x \qquad \text{for } 10 \leq x \leq 100$$
$$pdf(x) = 0 \qquad \qquad \text{elsewhere}$$

where A is a normalization constant. Find the value of A that makes pdf(x) a valid probability density function.

2) Almost all programming languages have a function to generate uniformly distributed random numbers (usually between 0 and 1). However, it's often useful to have normally distributed random numbers instead. Using the Box-Muller Transform, write some code to generate a standard normal distribution (mean of 0, standard deviation of 1.0) from uniformly distributed random numbers. Now generate 5000 normally distributed random numbers with a mean of 5.0 and standard deviation of 2.0. Plot a histogram of your distribution to check that you did this correctly.

3) You will often have to plot multivariable functions. For example, consider the following pdf, f(x,y) which is a 2-dimensional normal distribution where x and y are independent of one another:

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\left(\frac{x - \mu_x}{\sigma_x}\right)^2 - \left(\frac{y - \mu_y}{\sigma_y}\right)^2 \right]$$

Plot the pdf for the case where $\mu_x = \mu_y = 0$ and $\sigma_x = \sigma_y = 1$. That is, the x- and y-axes are the values for x and y, while the third dimension shows the value of the probability of getting these values for x and y. Does it appear as you expected?

However, 3-D plots are hard to do, so in this course, you can best show this distribution using a colourmap/heatmap or contour plot.

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1. A valid probability density function must sum up to $\underline{1}$.

So,

$$\int_{-\infty}^{\infty} pdf(x)\,dx = \underline{1} \implies \int_{10}^{100} \frac{A}{x}\,dx = \underline{1}$$

since $pdf(x) = 0$ when x not in $[10, 100]$

Solve for A:

$$A \cdot \int_{10}^{100} \frac{dx}{x} = \underline{1} \qquad \text{integral of } \int 1/x = \ln(x) + const.$$

$$A \cdot \left[\ln(100) - \ln(10) \right] = \underline{1} \qquad \ln(100) - \ln(10) = \ln\left(\frac{100}{10}\right)$$

$$A \cdot \ln(10) = \underline{1}$$

$$\boxed{A = \frac{1}{\ln(10)} \sim 0.434}$$

Notes: In a calculation-type problem like this, I would mostly be looking for you to:
1) Set up the problem to indicate you understand what you want to do (i.e. stating that the integral of the pdf must equal 1)
2) Write the equations out for this specific problem (i.e. the second line of math)
3) Provide notes to explain why you are doing these steps
4) Show enough work so that I can follow your math and give partial credit if you have the right idea but made a math mistake (in this example, I was extra explicit but it's better to have a bit extra than too little). Also, you might want to go back to these calculations in the future so you might want to make sure you can follow your own work too!

2. If U_1 and U_2 are uniformly distributed random variables between 0 and 1, then using the Box-Muller Transformation, we can get:

$$N_1 = \sqrt{-2 \ln(U_1)} \, \cos(2\pi U_2)$$

$$N_2 = \sqrt{-2 \ln(U_1)} \, \sin(2\pi U_2)$$

where N_1 and N_2 are normally distributed random variables with mean $=0.0$ and standard deviation 1.0.

[source: mathworld.wolfram.com/Box-MullerTransformation.html]

I used MATLAB's rand() function to generate 1000 uniformly distributed random numbers and then used the above transformation to get 1000 numbers distributed according to the standard normal distribution.
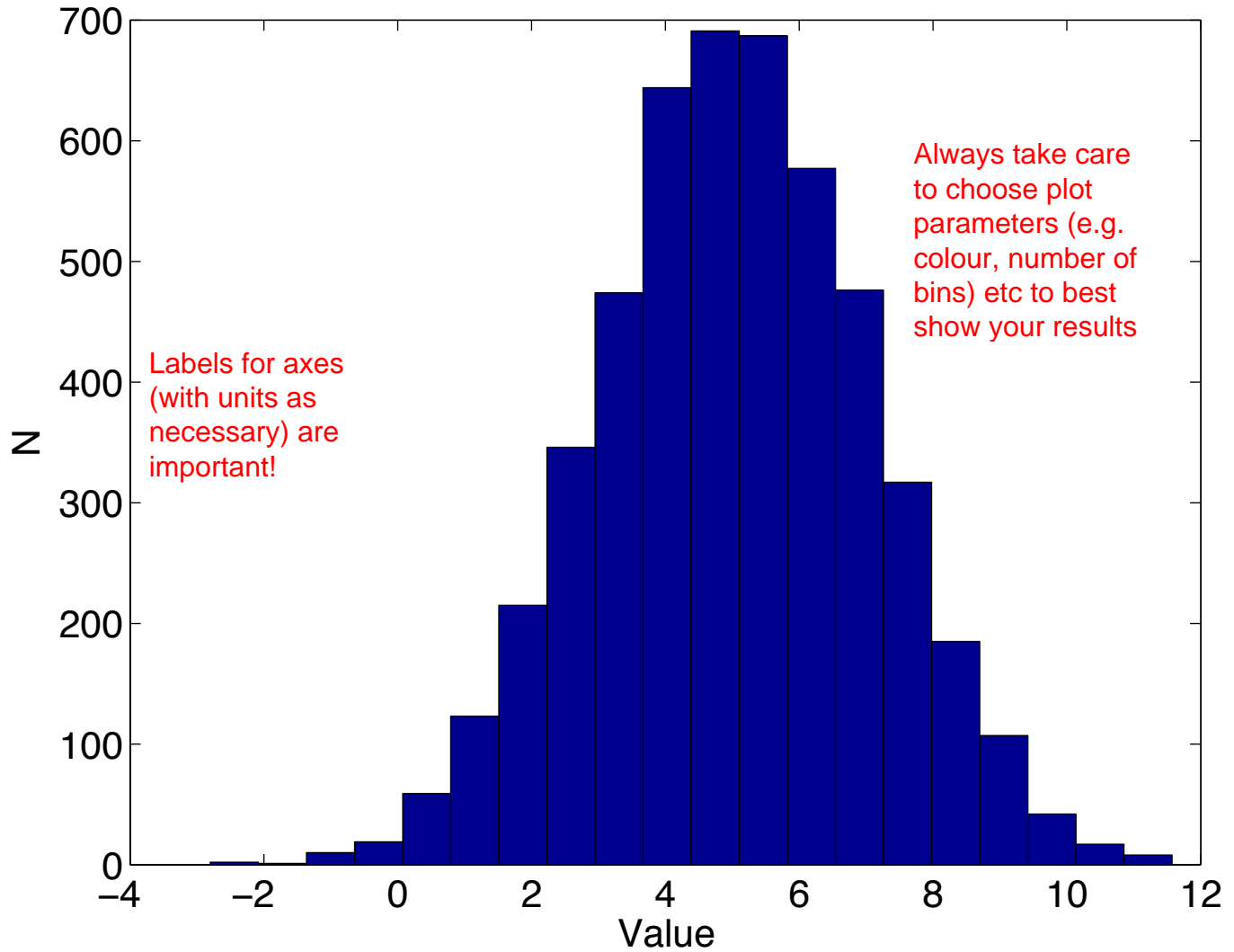
In order to get the required distribution (normal with mean $= 5$ and standard deviation $= 2$), I multiplied my generated standard normal random numbers by 2 and then added 5.

Figure 1 is a histogram of this distribution, using 20 bins. The shape appears normally distributed with $\mu = 5$, $\sigma \approx 2$.

Figure 1: Normal distribution from Box−Muller Transform

Having a figure label and some short description is helpful!

Always take care to choose plot parameters (e.g. colour, number of bins) etc to best show your results

Labels for axes (with units as necessary) are important!

Scale your ranges to best show your data
Also, make sure your font sizes are legible!

3. Figure 2 is a colourmap of a 2D normal distribution.

$$f(x,y) = \frac{1}{2\pi} e^{-x^2-y^2}$$

I plotted it in the range $-5 \leq x \leq +5$, $-5 \leq y \leq +5$ which captures most of the probability density.

The distribution is symmetrical and does indeed appear to be a 2D normal distribution.

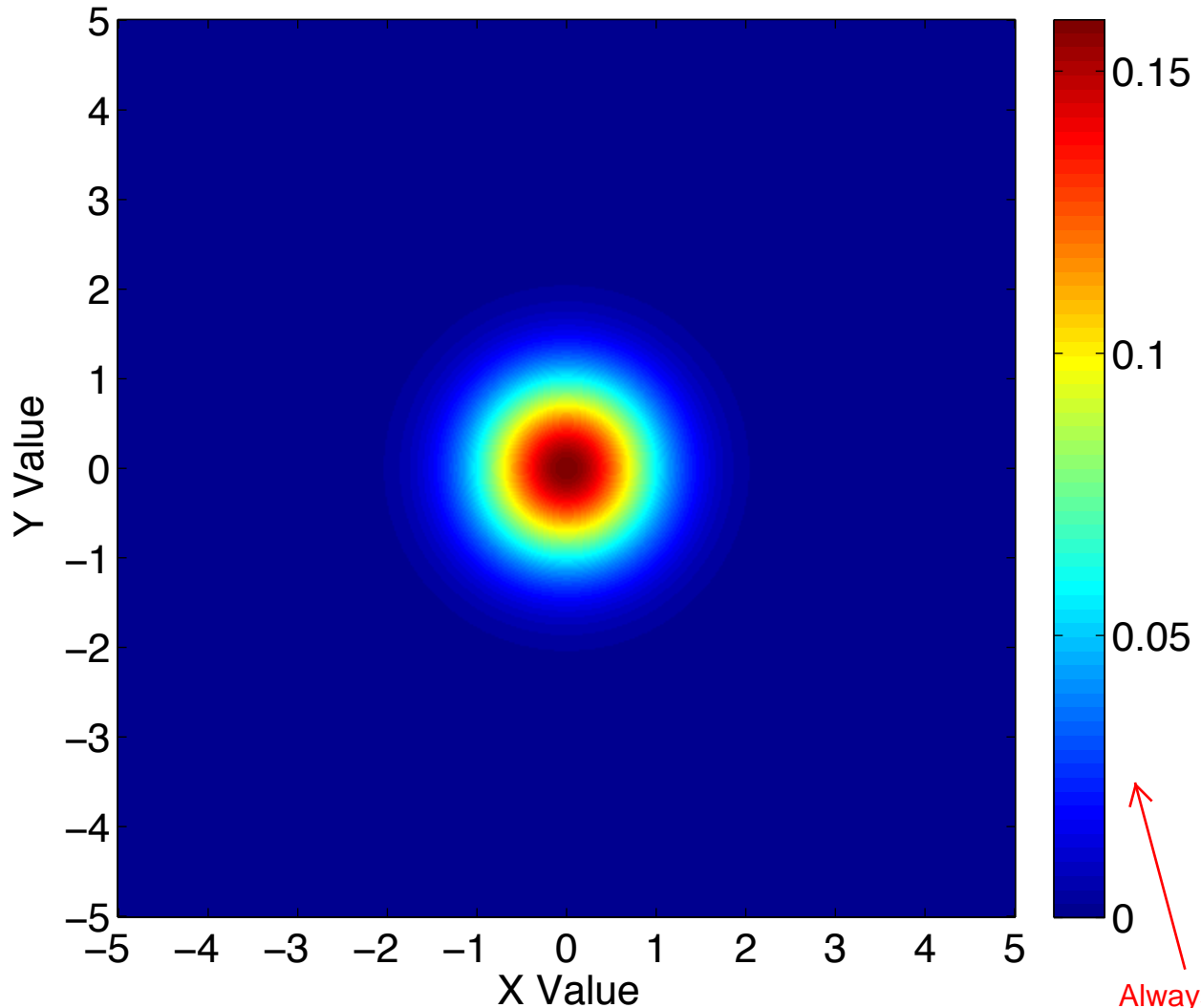It's helpful to describe what you are plotting!

this is better described as a probabtility density

Figure 2: 2–D Normal distribution

Choose a plot range that shows what you want to show! Sometimes, it might make sense to show an overview plot and then zoom in on some regions to show detail (as separate plots or as insets, whatever is easier for you)

Always include a colourbar for these types of plots, and indicate what value is being plotted, either as a colourbar title (you can just write it in) or as part of the plot title, as corrected