Ph 20.7 - Linear and non-linear regression: the Hubble plot using Type-1a supernovae

Due: Week 10 -v20190220-

Introduction - Hubble's Law and the cosmological constant

In this assignment you will develop and exercise your python curve-fitting skills, using some interesting data downloaded from the web.

We will look at some data that establishes the *acceleration of the expansion of the universe*. If you are not familiar with modern cosmology, give yourself a crash course by reading the Wikipedia articles for all of the technical terms in italics, below.

One of the pillars of modern *cosmology* is *Hubble's Law*, which relates an object's *cosmological redshift* (z) to its *luminosity distance* (d_L). From such a plot, we can directly measure the expansion rate of the universe, in terms of the *Hubble parameter* $H(z)$.

For small redshifts (say, $z < 0.05$), this tests the linear relation $d_L = cz/H_0$, where c is the speed of light and H_0 is the *local Hubble parameter* or the *Hubble constant*. H_0 is roughly 70 km/s/Mpc, where Mpc $= 10^6$ *parsecs*; one parsec is 3.26 light-years or 3.0857×10^{16} m.

For larger redshifts, $H(z)$ tells us whether the rate of expansion of the universe is *accelerating* or *decelerating*; the Hubble Law becomes non-linear. The fact that the gravitational interaction of matter is always attractive means that all of the matter in the universe should slow the expansion: we expect the *deceleration parameter* q to be greater than zero.

In astronomy, measuring the redshift of atomic lines observed in the spectrum is relatively easy. It is hard to measure distances. However, if an event (such as a *Type 1a supernova* explosion) emits a known amount of electromagnetic energy, it is a *standard candle*. By measuring the event's apparent brightness, one can thus infer its luminosity distance. Although type-1a supernova (which are very bright and can be detected even at cosmological distance of Gpc and beyond) are not standard candles, they appear to be *standardizable candles*: by measuring the rise and fall of their luminosity over time (their *light curve*), one can infer their absolute luminosity via the so-called *Phillips relationship*.

In the late 1990's several groups measured the luminosity distance and redshift of many high-redshift type 1a supernovae, tested the Phillips relationship, constructed a Hubble plot, and inferred the Hubble constant H_0 and deceleration parameter q.

They went on to fit the curve in terms of the prediction from General Relativity of the expansion history of the universe: the *Friedman-Lemaitre-Robertson-Walker FLRW metric*. They fit their data to that prediction, in terms of three parameters: the total density of matter (ordinary matter and *dark matter*) ρ_m, relative to the *critical energy density of the universe* $\rho_c = 3H_0^2/8\pi G$, so $\Omega_m \equiv \rho_m/\rho_c$; the density associated with a cosmological constant Ω_Λ, and the curvature of the universe Ω_k; with the constraint $\Omega_m + \Omega_\Lambda + \Omega_k = 1$.

The analyses of those data showed the opposite of what they were expecting: the deceleration parameter $q < 0$, or, equivalently, $\Omega_\Lambda > 0$. The expansion of the universe appears to be *accelerating*, not decelerating, due to some hypothesized *dark energy*; the simplest version is the *cosmological constant* Λ.

After much cross-checking, they convinced most astronomers and cosmologists that this was a real effect. Since then, the evidence has grown. This astonishing result suggested that there was something, not yet understood, which governs the expansion of the universe at large distance scales. This discovery led to the *2011 Nobel Prise in Physics*.

This Assignment

In this assignment, you will download their data, fit it to the deceleration and FLRW metric prediction, and (presumably) arrive at the same result.

1. There are several groups who have released data from low- and high-redshift Type 1a supernovae. You can choose your favorite; but let's start with the Supernova Cosmology Probe's Union 2.1 Compilation from 2011: `http://supernova.lbl.gov/Union/` . Download the data as a txt file. Browse it. Read it in to python.

2. The data for each SN is given in terms of *distance modulus*, which you should convert to luminosity distance in units of Mpc. Ditto for the uncertainty on the distance modulus.

3. Plot these data on a linear-linear scale, both for (distance modulus vs redshift), and (luminosity distance vs redshift).

4. Select low-redshift events (eg, $z < 0.05$). Assume they follow a linear Hubble law: $d_L = cz/H_0$, and compute H_0. Plot this on top of the data; clearly, higher-redshift events are in the non-linear regime.

5. Fit the data to a non-linear Hubble's Law including a deceleration parameter q: $d_L = cz/H_0(1 + ((1 - q)/2)z)$. You can try this separately for the luminosity distance and for the distance modulus; the results will be pretty similar. Plot the fit results on top of the data, and also plot the normalized residuals in a separate plot underneath.

   - There are lots of ways to do these fits.
   - You can start in one dimension, assuming $H_0 = 70$ and stepping through values of q, constructing the χ^2 versus q and looking for the minimum.
   - You can do it in 2 dimensions, plotting the χ^2 in a 2D grid of H_0 and q.
   - You can linearize the problem by choosing to parameterize in terms of $1/H_0$, then do 2D linear regression by constructing 2D matrices and then using `np.linalg.solve`.
   - You can use non-linear regression by using `scipy.optimize.curve_fit`.
   - You are strongly urged to use both of the above methods, to build your skill-set.
   - If you want to get fancier (totally optional!), you can anticipate Ph 21 by using Bayesian parameter estimation, with `dynesty`: `https://dynesty.readthedocs.io/en/latest/index.html`

6. Instead, use the FLRW metric expression for the luminosity distance, as given in the Wikipedia article on distance measures in cosmology. Here, your parameters are H_0 and Ω_m, with $\Omega_\Lambda = 1 - \Omega_M$ and $\Omega_k = 0$: a flat universe, as is well established from the *Cosmological Microwave Background* (CMB). Make all the same plots and do all the same fits.

7. What is the *statistical significance* of the statement that $\Omega_\Lambda > 0$, ie, that the expansion of the universe is accelerating?

8. You are welcome to explore the criticisms and limitations of this analysis and conclusion.

9. Latest data from quasars have been added to the mix. If you are interested, the paper from 2019 is at `http://www.pmaweb.caltech.edu/~physlab/Hubble/Risaliti_et_al-2019-Nature_Astronomy.pdf` and the data are at `http://www.pmaweb.caltech.edu/~physlab/Hubble/DL_all_short.txt` .